A time consistent waveform inversion (TWIN) method

Introduction

The pioneering works of Lailly (1983) and Tarantola (1984) on Full-Waveform inversion (FWI) method raised the hope of effectively interpreting all recorded seismic events. Unfortunately, it is now well established (e.g., Virieux and Operto (2009)) that conventional FWI has strong limitations for updating the long-wavelength component of velocity models (background velocity model) from reflected data only. To handle this problem, Reflection FWI (RFWI) approaches propose to split the FWI model space into “reflecting” and “propagating” parts (see for instance Xu et al. (2012), Clement et al. (2001), Zhou et al. (2015)). Some approaches (referred as RWI approaches hereafter) consider a Born approximation based de-migration modeling as formulated by Chavent et al. (1994) in frame of the MBTT method and later revisited by Xu et al. (2012), Brossier et al. (2015) and Kryvohuz et al. (2019), among others. To model the (primary) reflected events $d^{\text{cal}}$, they use a wave-equation based Born modeling operator $B(m_b)m_r$ depending (linearly) on the reflectivity $m_r$ and (non-linearly) on background model $m_b$.

A reflectivity model $m_r(m_b)$ consistent with a background model $m_b$, may be obtained as the solution of the following quantitative migration problem:

$$m_r(m_b) = \arg \min_{m_r} \mathcal{C}_r(d^\text{cal}_\text{mig}(m_b,m_r), d^\text{obs}_\text{mig}) \text{, with } d^\text{cal}_\text{mig}(m_b,m_r) = B_{\text{mig}}(m_b)m_r$$

(1)

Note that the observed “migration” data $d^\text{obs}_\text{mig}$ used for the migration problem (Eq. (1)) may be a subset of the observed data $d^\text{obs}$ (for instance the short-offset data), $B_{\text{mig}}$ being the de-migration operator and $\mathcal{C}_r$ being usually a least-squares criterion. For convenience, we symbolically define a migration operator $\text{Mig}(m_b)$ such that the reflectivity model $m_r(m_b) = \text{Mig}(m_b) d^\text{obs}_\text{mig}$ is solution of the migration variational problem (1).

Each iteration of a conventional RWI optimization scheme (Xu et al. (2012)) consists in updating sequentially the reflectivity and the background models. For a given iteration “$k$” considering a provided background model $m^{k-1}_b$ and the corresponding reflectivity model $m^{k-1}_r = \text{Mig}(m^{k-1}_b)d^{\text{obs}}_{\text{mig}}$, the updated background model $m^{k}_b$ is obtained by optimizing:

$$m^{k}_b = \arg \min_{m^{k}_b} \mathcal{C}_b(d^{\text{cal}}_{\text{rw}i}(m^{k}_b,m^{k-1}_r), d^{\text{obs}}_{\text{rw}i}) \text{, with } d^{\text{cal}}_{\text{rw}i}(m^{k}_b,m^{k-1}_r) = B_{\text{rw}i}(m^{k}_b)m^{k-1}_r$$

(2)

The data $d^{\text{obs}}_{\text{rw}i}$ (referred as “RWI data”) may also be a subset of the observed data (for instance long-offset data). Therefore, Born modeling operators $B_{\text{mig}}$ (Eq. (1)) and $B_{\text{rw}i}$ (Eq. (2)) only differ by their output spaces (corresponding to the migration and RWI data-sets, respectively). As discussed and illustrated by Valensi et al. (2017) and Baina and Valensi (2018), the fact that the reflectivity model $m^{k-1}_r$ in Eq. (2) is fixed when estimating the background model creates an inconsistency between the background and reflectivity models. This inconsistency leads to a biased estimated model $m^{k}_b$ and a slow convergence of the overall optimization scheme. Also, there is no guarantee that the RWI data misfit decreases for the updated background model $m^{k}_b$ when the reflectivity is re-estimated with that background model. These reasons motivated introducing the concept of Reflectivity-Velocity Consistent (RVC) RWI schemes (Baina and Valensi (2018), Valensi and Baina (2019)), where the consistency between reflectivity and background models is enforced through systematic reflectivity estimation, which results a reduced misfit functional (instead of optimizing sequentially Eqs. (1) and (2)):

$$m^{k}_b = \arg \min_{m^{k}_b} \mathcal{C}_b(d^{\text{cal}}_{\text{rcv}r\text{w}i}(m^{k}_b), d^{\text{obs}}_{\text{rw}i}) \text{, with } d^{\text{cal}}_{\text{rcv}r\text{w}i}(m^{k}_b) = B_{\text{rw}i}(m^{k}_b)\text{Mig}(m^{k}_b)d^{\text{obs}}_{\text{mig}}$$

(3)

Note that the MBTT method (Chavent et al. (1994)) and RWI variable projection approaches (Huang and Symes (2015)) may be considered as particular instances of RVC schemes.

A Time-consistent Wave-form INversion (TWIN) method

A migration minimal data-set (Padhi and Holley (1997)) is the minimal sub-set of the data ensuring that the migration and de-migration operations are unique and well-defined for any (acceptable) velocity model. A candidate of migration minimal data-set is the Zero-Offset (ZO) data $d^{\text{obs}}_{zo}$. We define $B_{zo}$ as the zero-offset Born-modeling operator and $B^{-1}_{zo}$ as the inverse operator (ZO migration) verifying:

$$d^{\text{obs}}_{zo} = B_{zo}(m_b)B^{-1}_{zo}(m_b)d^{\text{obs}}_{zo}$$

(4)
for any acceptable zero-offset data $d_{zo}^{obs}$ and velocity model $m_b$. We formulate the TWIN as a RVC-RWI scheme based on a ZO migration to estimate the reflectivity $m_r(m_b) = B_{zo}^{-1}(m_b) d_{zo}^{obs}$, which results in the following optimization problem:

$$m_b^* = \arg \min_{m_b} \mathcal{E}_b(d_{twin}^{cal}(m_b), d_{twin}^{obs}) \text{, with } d_{twin}^{cal}(m_b) = B_{rw}(m_b)B_{zo}^{-1}(m_b)d_{zo}^{obs}$$

(5)

Note that expression (4) corresponds to a zero-offset invariance condition ensuring the consistency between observed and calculated data during the inversion process. After differentiation of Eq. (4) with respect to $m_b$, we obtain:

$$\mathcal{D}_m B_{zo}^{-1}(m_b) d_{zo}^{obs} = -B_{zo}^{-1}(m_b) \mathcal{D}_m (B_{zo}(m_b)) B_{zo}^{-1}(m_b) d_{zo}^{obs}$$

(6)

Differentiating the misfit function in Eq. (5) with respect to $m_b$ (application of chain, products rules of derivation and using Eq. (6)), we obtain the following misfit function gradient:

$$\nabla_{m_b} \mathcal{E}_b \left( d_{twin}^{cal}(m_b), d_{twin}^{obs} \right) = \left[ \mathcal{D}_m (B_{zo}(m_b)) B_{zo}^{-1} (m_b) d_{twin}^{obs} \right] \mathcal{F} \left( d_{twin}^{cal} \right) - \left[ \mathcal{D}_m (B_{zo}(m_b)) B_{zo}^{-1} (m_b) d_{twin}^{obs} \right] \mathcal{F} \left( d_{twin}^{cal} \right)$$

(7)

with the “adjoint-source” $s(d_{twin}^{cal})$ defined as $s(d_{twin}^{cal}) = \frac{\delta \mathcal{E}_b(d_{twin}^{cal}, d_{twin}^{obs})}{\delta d_{twin}^{cal}}$ and the symbol $\mathcal{D}_m$ applied to an operator $F(m_b)$ indicating the tangent application (Fréchet derivative): $\mathcal{D}_m F(m_b) \delta m_b = F(m_b + \delta m_b) - F(m_b) + O(\|\delta m_b\|^2)$ and the superscript $\dagger$ indicating the adjoint operator.

We can identify the first term of the left-hand-side in Eq. (7) as the conventional RWI gradient (Xu et al. (2012)) and a supplementary Reflectivity-Velocity Consistency (RVC) term resulting from the coupling between reflectivity and background models. In figure 1c, the TWIN gradient is depicted for an homogeneous background model and a plane reflector (single source-receiver pair, L2 misfit function). This gradient is a sum of a conventional RWI gradient (Fig.1a) plus the RVC term (Fig.1b). The conventional gradient term may be interpreted as projection of the adjoint source onto the reflected wavefield sensitivity kernel (“wave-paths”). The RVC term may be interpreted as the projection the adjoint source onto the sensitivity kernel due to the reflectivity variations generated by the background model perturbations. Therefore, the conventional RWI approach attempts to update the background model considering only the reflected “wave-paths” whereas the TWIN approach also accounts for the effect of the background model perturbation to reflectors “displacements” (reflectivity perturbations) by updating along the zero-offset “wave-paths”. Note that the signs of the two gradient terms (Figs.1a and 1b) are opposite, so that in case of collocated sources and receivers, they destructively interfere (which is consistent with the fact that ZO data alone do not constrain the background velocity). Therefore, we expect that the RVC term in many instances has a magnitude comparable to the fixed reflectivity term (conventional RWI gradient), meaning that RVC gradient term can not be considered as negligible.

**Numerical experiment: comparison on the Chevron benchmark model**

To compare TWIN to alternative ways of handling the RVC effect, we propose an experiment based on the Chevron SEG 2014 benchmark. The initial model is constructed from a previously inverted model...
where we roughly draw an horizon below the low-velocity layer (about 3.5 km depth), followed by a linear velocity extrapolation up to 4800 m/s. The resulting model (Fig. 2a) is kinematically incorrect in depth as confirmed by the RTM-offset gathers (Fig.3a). Data are filtered (filter shape: 2-3-12-15Hz) and linearly muted to remove direct and refracted events. This experiment consists of two tests. The first one is a “conventional RWI” inversion (where the reflectivity and background are sequentially estimated) and the second test consists of using the TWIN approach. In each case, 30 iterations are performed with a gradient smoothing of 250m vertical and 1000m lateral lengths, all other parameters are set to the same values. The result from conventional (sequential) RWI (Fig.2b) approach suffers from vertically-extended high-velocity artefacts (“vertical chimneys”) below 3.5 km depth which strongly distort the reflectivity below 4km depth. This may be confirmed by the associated RTM-offset gathers (Fig. 3b) where we can see a significant remaining residual move-out below 4km depth. Note that even if we would perform more iterations, these artefacts will not be significantly corrected. The model obtained with the TWIN approach (Fig. 3c) does not suffer from the “vertical chimney” artefacts and reflectors in depth are much flatter. We also observe that even the shallow part of the model are significantly updated with TWIN. The accuracy of TWIN model may be assessed with the RTM-offset gathers depicted in figure 3c, where most of the initial residual move-out is corrected.

Concluding remarks
To emphasize the conceptual similarities between our approach and the (ray-based) reflection tomography method (e.g. van Trier (1989)) using the concept of zero-offset travel-times invariants to handle the velocity-depth coupling, we named our approach the “Time-consistent Waveform INversion” (TWIN) method. Compared to the conventional RWI approach (sequentially updating reflectivity and background models), the consistency between these two models is constantly enforced during the inversion. The TWIN misfit function gradient also contains a supplementary term to account for the reflectivity-background coupling effects. Generally, the TWIN convergence is faster than conventional RWI and inversion results are superior (in particular, they are less susceptible to vertically-extended artefacts in the deep parts of the model). Finally, we emphasise a difference between the TWIN approach and the proposals of Snieder et al. (1989), Hicks and Pratt (2001), Plessix (2013) and Brossier et al. (2015), who re-parametrize the depth axis of the models in terms of vertical time (to maintain an approximate kinematic invariance at zero-offset): whereas our approach is general, the vertical-time transformation relies on strong assumptions of lateral invariance of velocity and reflectivity models.

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Figure 3: RTM offset gathers obtained with initial and final inverted models (using the conventional RWI and TWIN approaches). The green arrows indicate some reflectors with significant residual move-out in Figs. 3a and 3b, and mostly flattened in Fig. 3c.

References


