Introduction

Geostatistics is a significant technology whose objective is characterising the complex geological features. In the earth sciences, geostatistics has been applied to address numerous challenging tasks, such as modeling the subsurface elastic properties. Generally, geostatistics can be divided into two types, which are two-point geostatistics (TPS) and multiple-point geostatistics (MPS).

The TPS approaches, such as kriging interpolation and sequential Gaussian simulation, have been applied in geological modeling and seismic inversion (Journel, 1986). However, the TPS is incapable to depict the complex geological structures since the variogram in TPS theory merely represents the two-point correlation in the space. By comparison, MPS offers an alternative to characterise complicated geological structures by going beyond classical two-point statistics (González et al., 2007; Mariethoz et al., 2010). Certain MPS algorithms, such as the single-normal equation simulation (SNESIM) (Strebelle, 2002), have been developed. To deal with the heavy memory and calculation burdens of the MPS algorithms, numerous studies have been conducted. For example, Hansen and Bach (2016) utilized extended normal equations simulation (ENESIM) algorithm to improve the computational efficiency of MPS.

Currently, TPS can be both used in characterising the continuous variables like velocity and porosity and categorized variables like lithofacies, while the MPS algorithms are more suitable for the simulation of the categorized variables. Since each of MPS and TPS methods has its own advantages and shortcomings, the combination of MPS and TPS may be a reasonable choice. By combining the MPS and TPS approaches, we propose a geostatistical interpolation method for modeling the continuous elastic parameters. In the proposed method, we acquire the probability distribution of the lithofacies by the ENESIM algorithm. Based on the kriging interpolation, we obtain the velocity model from well data constrained by the lithofacies distribution. The proposed approach is valuable not only in geological modelling but also in building initial models for seismic inversion.

Method

Acquisition of lithofacies probability distribution

In ENESIM algorithm, the area to be simulated is discretized into a simulation grid with numerous nodes. At one unknown node $u$ of the simulation grid, a data event $\text{dev}(u)$, which includes several conditioning nodes around the unknown node, can be constructed by the data template. Then, by scanning the training image, we find the data events in the training image similar to the data event of the unknown node in the simulation grid. We assume that there are $n$ kinds of lithofacies in the simulation area. The probability of the $i$th kind of lithofacies at the unknown node can be expressed as,

$$
    p_i \approx \frac{c_i[\text{dev}(u)]}{c[\text{dev}(u)]},
$$

where $c[\text{dev}(u)]$ is the repeat count of data events in the training image, $c_i[\text{dev}(u)]$ is the repeat count of data events in which the lithofacies of central nodes is the $i$th one. By equation (1), we can obtain the probability distributions of the lithofacies at the unknown node $u$,

$$
    P = (p_1, p_2, \ldots, p_n), \quad \sum_{i=1}^{n} p_i = 1.
$$

Interpolation constrained by lithofacies probability distribution

We modify kriging interpolation method by the constraint of the probability distribution of the lithofacies. Considering a 2D profile with several wells, we utilize the well-log data to estimate the values of the elastic parameters, such as velocity and density. We assume that there are $k$ conditional nodes $x_1, x_2, \ldots, x_k$ around the unknown node $x$. The elastic parameter values of these nodes are
Based on equations (8) and (9), we present the multivariate covariance matrix for the lithofacies, respectively. The lithofacies probability distributions of these conditional nodes are expressed as,

\[
P(x_1), P(x_2), \cdots, P(x_k),
\]

\[
P(x_j) = [P_1(x_j), P_2(x_j), \cdots, P_n(x_j)], \quad \sum_{i=1}^{n} P_i(x_j) = 1.
\]

(3)

The lithofacies probability distribution of the unknown node \( x \) is \( P(x) \). For any two nodes \( (x_j, x_i) \) in the \( k \) conditional nodes, the probability of the two different nodes in the simulation grid with the \( i^{th} \) lithofacies concurrently can be represented as,

\[
P_{same,i}(x_j, x_i) = P_i(x_j) P_i(x_i), (i = 1, 2, \cdots, n),
\]

(4)

the probability of the two different nodes having different lithofacies is,

\[
P_{diff}(x_j, x_i) = 1 - \sum_{i=1}^{n} P_i(x_j) P_i(x_i).
\]

(5)

From the well-log data, we obtain the variograms of the \( n \) kinds lithofacies between \( x_j \) and \( x_i \) and express them as,

\[
\gamma = [\gamma_1(h), \gamma_2(h), \cdots, \gamma_i(h), \cdots, \gamma_n(h)],
\]

(6)

where \( \gamma_i(h) \) represents the variogram of the well-log data with the \( i^{th} \) lithofacies, \( h \) is the Euclidean distance between \( x_j \) and \( x_i \). According to the relationship between the variogram and covariance, the covariance of the \( i^{th} \) lithofacies between \( x_j \) and \( x_i \) can be calculated by,

\[
C_i(h) = \gamma_{\max,i} - \gamma_i(h),
\]

(7)

where \( \gamma_{\max,i} \) is the sill of the variogram of the \( i^{th} \) lithofacies. The covariances for the \( n \) kinds of lithofacies are \( C_1(h), C_2(h), \cdots, C_n(h) \).

Based on equations (4) and (5), we can express the covariance between \( x_j \) and \( x_i \) constrained by lithofacies as,

\[
C_{MPS}(h) = \sum_{i=1}^{n} C_i(h) P_{same,i}(x_j, x_i) + C_{diff}(h) P_{diff}(x_j, x_i),
\]

(8)

where \( C_{diff}(h) \) denotes the covariance between \( x_j \) and \( x_i \) when they have different lithofacies. \( C_{diff}(h) \) is also obtained from the well-log data by the statistical fitting. As the same manner, we also obtain the lithofacies-constrained covariance between the unknown node \( x \) and a conditional node \( x_j \),

\[
C'_{MPS}(h) = \sum_{i=1}^{n} C_i(h) P_{same,i}(x_j, x) + C_{diff}(h) P_{diff}(x_j, x).
\]

(9)

Based on equations (8) and (9), we present the multivariate covariance matrix for the \( k \) conditional nodes as,

\[
C_{MPS} = \begin{bmatrix}
C_{MPS}(0) & C_{MPS}(1) & \cdots & C_{MPS}(|x_k - x_1|) \\
C_{MPS}(1) & C_{MPS}(0) & \cdots & C_{MPS}(|x_k - x_2|) \\
\vdots & \vdots & \ddots & \vdots \\
C_{MPS}(|x_k - x_1|) & C_{MPS}(|x_k - x_2|) & \cdots & C_{MPS}(0)
\end{bmatrix}.
\]

(10)

The covariance vector between the unknown node \( x \) and the \( k \) conditional nodes can also be expressed as,

\[
C'_{MPS} = [C_{MPS}(|x - x_1|), C_{MPS}(|x - x_2|), \cdots, C_{MPS}(|x - x_k|)]^T.
\]

(11)

In the proposed interpolation method, the weight coefficients of the \( k \) conditional nodes can be calculated by,

\[
\lambda = C_{MPS}^{-1} C'_{MPS} = (\lambda_1, \lambda_2, \cdots, \lambda_j, \cdots, \lambda_k)^T.
\]

(12)
where $\lambda_j$ is the weight coefficient of the $j^{th}$ conditional node. Then, the value of the elastic parameter at the unknown node is expressed as,

$$M(x) = \sum_{j=1}^{k} \lambda_j M(x_j).$$

In the same way, we can estimate the model parameters of the entire profile.

**Examples**

We test the proposed method on a P-wave velocity model, which is a 2D profile of the Stanford VI-E reservoir models. Figures 1a and 1b display the P-wave velocity model and the corresponding lithofacies model. There are two kinds of facies in Figure 1b, which are the sandstone (in yellow) and shale (in blue). We use five pseudo wells for the test of the proposed method. The white lines in Figure 1 represent the locations of the pseudo wells. Moreover, we utilize another 4 profiles from the Stanford VI-E lithofacies model as the training image. Figure 2 shows the training image.

**Figure 1** The 2D true model of (a) P-wave velocity, and (b) lithofacies. The sandstone and shale are denoted by yellow and blue in (b). The white lines represent the locations of the pseudo wells.

**Figure 2** The training image obtained from the 3D Stanford VI-E lithofacies model

**Figure 3** (a) A single simulation result, and (b) the probability distribution of lithofacies obtained by ENESIM algorithm.

Using the above data, we conduct the MPS simulation by ENESIM algorithm. Figures 3a and 3b display a single simulation result and the probability distribution of lithofacies. Compared with the true lithofacies model shown in Figure 1b, the single simulation result is stochastic, while the probability distribution of lithofacies is able to represent the general distributions of sandstone and shale.
We then obtain the well-log data to estimate the P-wave velocity by the proposed interpolation method and display the interpolation result in Figure 4a. The result shown in Figure 4a keeps a similar geological structure with the true model shown in Figure 4a even some details are blurry. We also obtain the interpolation result by kriging method, and display the corresponding result in Figure 4b. Compared with the interpolation results shown in Figure 4a, the result of kriging method is so smooth that it is incapable to recover the geological bodies, which illustrates that the combination of MPS and TPS contributes to improving the geostatistical modeling effect.

**Figure 4** The interpolation result of P-wave velocity obtained by (a) the proposed method, and (b) kriging method.

**Conclusions**

By integrating MPS with TPS, we propose an interpolation method constrained by lithofacies for estimating the continuous subsurface variables. In the proposed method, the calculation of the covariance matrix, which is crucial in kriging interpolation, is constrained by the probability distribution of lithofacies obtained by MPS. We test the proposed method by estimating the P-wave velocity. The test results illustrate the effectiveness of the method. This work is a contribution to both geostatistics and seismic inversion. However, the acquisition of the training image may be an intractable issue in practice.

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**References**


