Introduction

Due to the limitations of seismic acquisition, including complicated geological conditions and construction. Incomplete seismic data has an impact on the subsequent date processing. Therefore, high quality of seismic data reconstruction is very important to obtain complete and regular data for seismic data processing and inversion.

In recent years, David Donoho proposed the compressive sensing theory, which replaces the conventional sampling and reconstruction operations with a more general linear measurement scheme coupled with an optimization in order to acquire sparse signals at a rate significantly below Nyquist. Suppose that the desired signal has a sparse representation in a known transform domain, then strong mathematics have shown that the original signal can still be determined uniquely from a small number of its acquired samples as long as the aliasing artefacts due to the sampling pattern are incoherent (noise-like) in that transform domain (Yang J L et al., 2018). However, seismic data are usually not sparse, so it is vital to find the appropriate transform, such as Fourier, Shearlet, Curvelet and Wavelet transform, which makes the signal coefficients sparse in this process. We propose to use Dost (Stockwell et al., 2007) as a new sparse transform method in the process of compressive sensing seismic data reconstruction. It is the first time that Dost is applied in compressive sensing. Dost has the good time-frequency analysis capability, it introduces three orthogonal basis vectors to represent the time-frequency space, which reduces the redundancy of S-transform and makes it have better sparsity (Yu Q et al., 2017).

In this paper, the principle of discrete orthogonal S transform and the theoretical framework of compressive sensing are expounded. We apply Dost as a new compressed sensing sparse transform method to achieve seismic data reconstruction, and combine the fast projection onto convex set (FPOCS) algorithm (Gan S et al., 2016) to obtain the reconstruction result. Quantitative evaluation have been done to the Dost, Fourier and Shearlet methods using three parameters (reconstruction time, error value, and signal to noise). Testing results of synthetic and real data verify the correctness and effectiveness of this method, which reconstruction accuracy is higher than that of Fourier and Shearlet transform.

Theory

The seismic data reconstruction problem is transformed into the form of mathematical equation through the compressive sensing theoretical framework:

$$ Y = \Phi x $$

where $X \in R_N$ specifies the complete seismic data under ideal conditions, $Y \in R_M$ represents the seismic data with missing trace actually collected, and $\Phi \in R_{M \times N} (M \ll N)$ is the sampling matrix.

Stockwell proposed a discrete orthogonal S transform method which can be defined as the inner products between a time series $h[kT]$ and the basis function $\Psi(kT)|_{\nu, \beta, \tau}$,

$$ S\{h[kT]\} = S\left( \tau, \nu, \beta \right) = \sum_{k=0}^{N-1} h[kT] \Psi(kT)|_{\nu, \beta, \tau} $$

on the basis, time-frequency space can be divided into N orthogonal basis vectors by the parameters $\nu$ is indicating the center of frequency variable, $\beta$ shows the width of the frequency band, and $T$ specifies a time variable. Each basis vector represents a different time-frequency block. In general, using appropriate parameters the Kth basis function is defined as:

$$ \Psi(kT)|_{\nu, \beta, \tau} = 1 / \sqrt{\beta} \sum_{k=-\beta}^{\beta} \exp \left( 2 \pi i \frac{f}{\beta} \right) \exp \left( 2 \pi i \frac{\tau}{\beta} \right) $$

where $1 / \sqrt{\beta}$ specifies normalization coefficient to ensure the basis function are directly orthogonal to each other.

The selection of three parameters has a great influence on the orthogonality of the basis function. In order to ensure the orthogonality of the basis function and simplify the assignment of the three parameters, the variable parameter $p$ is introduced and the specific assignment is $p > 1, \nu = 2^{(p-1)} + 2^{(p-2)}, \beta = 2^{(p-1)}, \tau = 0, 1, \ldots, 2^{(p-1)} - 1$.
For an input signal of arbitrary length $N$ generates $N^2$ coefficients by the discrete ST. It takes $O(N)$ times to calculate each coefficient, and the overall coefficients complexity is $O(N^3)$. However, the Discrete orthogonal S transform makes $N$ linearly independent points representing time-frequency signals to replace the time-frequency space in S transform, and Reduce the overall algorithm complexity to $O(N^2)$. Therefore, considering its low algorithm complexity and strong time-frequency resolution, we apply it to the sparse transform of compressed sensing.

In this paper, we introduce Dost to carry out sparse representation of seismic data, which can be obtained:

$$X = \Psi_{Dost}$$  

(4)

where, $\Psi_{Dost} = [\Psi_1, \Psi_2, \cdots, \Psi_N]$ is the basis vector of Dost $\theta = [\theta_1, \theta_2, \cdots, \theta_N]$, is the coefficient vector of seismic data based on Dost, and Vector $\theta$ has to satisfy $\|\theta\|_0 = K$.

Combining the Equation (1) and (4), the seismic data reconstruction problem can be write by:

$$\hat{\theta} = \arg\min_{\theta} \|\theta\|_0 \ s.t. Y$$

$$\Phi \Psi_{Dost} \theta$$

(5)

Considering the error interference in the actual processing, introducing $\lambda$ as the equilibrium operator of $L_1$ norm and $L_2$ norm,

$$\hat{\theta} = \arg\min_{\alpha} \|\alpha\|_1 + \|Y - \Phi \Psi_{Dost} \theta\|_2^2$$

(6)

In this paper, FPOCS algorithm is adopted to solve the CS data reconstruction problem in equation (7). FPOCS algorithm combines the advantage of POCS method high reconstruction precision and the FISTA method fast convergence speed. The algorithm is expressed as follows: Input is sampling matrix $\Phi$, missing seismic data $Y$, total number of iterations $K$, identity matrix $I$, and threshold function $T$. Output is the reconstruction of seismic data $X$. We set $t_0 = 1, X_0 = 0, k = 0$ at the beginning,

step 1: $t_{k+1} = 1 + \sqrt{1 + 4t_k^2}/2$

step 2: $\bar{X}_k = X_k + (X_k - X_{k-1})/t_{k+1}$

step 3: $X_{k-1} = Y + (I - \Phi)\Psi_{Dost}T_\lambda(\Psi_{Dost}^T\bar{X}_k)$

step 4: reduce $\lambda$ by threshold model, $i = i + 1$.

step 5: if $1 < K$, or not satisfy the convergence condition, return step1;otherwise, $\hat{X} = X_{k+1}$.

In order to compare the data reconstruction results, we introduced two evaluation parameters as follows:

(1) Error value ($E_r$),

$$E_r = \|\Phi \hat{\theta} - (Y - A \theta)\|_2$$  

(7)

(2) Signal to noise ratio (SNR),

$$\text{SNR} = 10 \log_{10} \left( \frac{\|Y\|_2^2}{\|X - \hat{X}\|_2^2} \right)$$

(8)

Examples

The synthetic seismic data of Fig. 1(b) is obtained by the forward model of Fig. 1(a), and Fig. 1(c) is the missing seismic data obtained by randomly missing 50%. Figure 1 (d–i) shows the reconstructed seismic data and difference results obtained by the three methods of Fourier, shearlet and Dost. By comparing Fig. 1 (g–i), it can be found that the result of Dost reconstruction is the best, while the result of Fourier introduces too much noise, and the result of Shearlet has some partial signal loss. By comparing Table 1, the reconstruction time of the Dost method is much lower than that of the Shearlet method, and the reconstruction effect is better than Shearlet and Fourier. We also get the same conclusion from the real seismic data to prove the effectiveness of the method.
**Figure 1** Synthetic seismic data: (a) the forward model, (b) complete data, (c) 50% random missing data. Reconstruction results of (d) Fourier, (e) Shearlet, (f) Dost, and residual profile of (g) Fourier, (h) Shearlet, (i) Dost in real seismic data.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Miss Rate</th>
<th>Iterations</th>
<th>Reconstruction Time(s)</th>
<th>Error Value</th>
<th>SNR(db)</th>
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</thead>
<tbody>
<tr>
<td>Dost</td>
<td>50%</td>
<td>30</td>
<td>45.292</td>
<td>0.108</td>
<td>17.4123</td>
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<tr>
<td>Fourier</td>
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<td>21.142</td>
<td>0.251</td>
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<tr>
<td>Shearlet</td>
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<td>30</td>
<td>1589.643</td>
<td>0.131</td>
<td>15.8014</td>
</tr>
</tbody>
</table>

**Conclusions**

In this paper, a seismic data reconstruction method based on discrete orthogonal S transform is proposed in the framework of compressed sensing theory. We apply Dost to compressed sensing for the first time, apply three orthogonal basis functions to construct sparse transform matrix, and combine FOOCS algorithm to optimize the overall reconstruction algorithm. The validity of the proposed method is proved by the synthetic seismic data and real seismic data.
**Figure 2** Real seismic data: (a) complete data, (b) 50% random missing data, Reconstruction results of (c) Fourier, (d) Shearlet, (e) Dost, and residual profile of (f) Fourier, (g) Shearlet, (h) Dost in real seismic data.

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**References**


