Introduction

Modeling tsunamis consist of two linked parts. First, defining the parameters of the source and calculation of the initial displacement of water (see Okada, 1985) for more information). Second, modeling the propagation of the initial water displacement in a prior defined region. When the potential energy of a tsunami-genetic source is converted to kinetic energy, initial water displacement is generated, which has an initial velocity and a sea surface deformation similar to the co-seismic deformation (Bolshakova, Nosov, & Kolesov, 2015). The velocity is high about hundreds of kilometers per hour, and the height is usually small (few centimeters). As the wave travels to the shore, due to dispersion, its velocity is decreased within a range of smaller velocities. By approaching the shore, the dispersed waves with lower velocity meet, and their amplitude are superimposed together. This phenomenon is called the shoaling effect and occurs when the depth of the sea becomes lesser than the wavelength (see Ward, 2011) for more information). The basis of tsunami modeling is to obtain governing equations and propagate the shallow water equations within the medium. It is referred to as shallow not because we are dealing with waves in shallow water, but since the depth of the ocean is small in comparison to the wavelength of the tsunami waves generated in the deep ocean.

The shallow water equations describe the governing equations of the basic hydrodynamic model for tsunami generation by disturbances on the surface ((Akylas, 1984; Nosov & Skachko, 2001)). This evolution equation derived from the theory of water waves, which approximate the behavior of real ocean, including coupled differential equations (Grilli, Guyenne, & Dias; Layton & van de Panne, 2002; Pelinovsky, Talipova, Kurkin, & Kharif, 2001)). First, the region is discretized into grid points, and by having the value of all grid points at the initial time and implementing discretization techniques such as Finite Difference Method (FDM), we try to model the propagation of a disturbance on other nodes. We are dealing with a partial differential equation, and we want to find the value of all variables at all times. The basis of these equations and implementations are shown in the Methodology section by using a second-order Runge-Kutta method (Butcher, 2007), which includes obtaining numerical approximations to the solution of initial value problem using the central difference scheme.

Besides the materials discussed above, one must note that solving a partial differential equation (PDE) for a tsunami in any region requires a calculation time (cost). The cost varies due to the accuracy of the round-off error, grid size, and the number of grid points. Therefore, for modeling any new scenario, a new time (cost) is required. Partial differential equations of wave motion are discretized using an explicit Finite Difference Method for our analysis. One could further extend and use other numerical finite difference discretization methods such as Crank Nicolson (Crank & Nicolson, 1947), Lax Wendorff (Lax & Wendroff, 1960) and Leapfrog scheme for modeling tsunamis (Goto, Ogawa, Shuto, & Imamura, 1997). In this manuscript, we seek to find out the effect of changes in the initial water displacement of tsunamis. In this approach, we have proposed a formula for the initial water displacement to make the computation more compatible. The effect of changing the parameters of the proposed formula was studied by performing several numerical modeling of the IWD’s in a predefined mesh grid medium.

Method and/or Theory

We propose Eq. (1) to represent a typical symmetric initial water displacement. The parameters $a$, $b$, and $c$ in Eq.(1) are scaling parameters in $z$, $x$, and $y$-direction respectively.

$$z_0 = a x e^{-\left(\frac{x^2}{b} + \frac{y^2}{c}\right)}$$

(1)

The value of $z_0$ in Eq.(1) can produce a scaled initial water displacement. By changing the scaling parameters, the height and skewness of the initial water displacement would vary. The increase in the values of parameters $a$, $b$, and $c$ results in an increase in $z$, $x$, and $y$-direction, respectively. Once the scaled initial water displacement is generated, one could use Eq.(2) to apply rotation and create a rotated initial water displacement. By changing the rotation parameter $-\theta$ in Eq.(2), the initial water displacement is rotated counterclockwise around the z-axis. In Eq.(1), the $X$ and $Y$ are values in the new grid after rotation, $x_0$, and $y_0$ are the values of $x$, and $y$ before rotation, and $R$ is the “Rotation Vector”.

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The initial water displacement, which has been generated with scaling and rotation parameters, is transferred and applied to the predefined modeling area mesh grid (See Figure 1). In the CFD modeling, the height and the velocity components are computed at all of the grid points at all of the time steps. A predefined time is considered to terminate the CFD process.

\[
\begin{bmatrix}
    x' \\
    y'
\end{bmatrix} = R \begin{bmatrix}
    x_0 \\
    y_0
\end{bmatrix} = \begin{bmatrix}
    \cos \theta_t & -\sin \theta_t \\
    \sin \theta_t & \cos \theta_t
\end{bmatrix} \begin{bmatrix}
    x_0 \\
    y_0
\end{bmatrix} = \begin{bmatrix}
    x_0 \cos \theta_t & -y_0 \sin \theta_t \\
    x_0 \sin \theta_t & y_0 \cos \theta_t
\end{bmatrix}
\]

(2)

The initial water displacement, which has been generated with scaling and rotation parameters, is transferred and applied to the predefined modeling area mesh grid (See Figure 1). In the CFD modeling, the height and the velocity components are computed at all of the grid points at all of the time steps. A predefined time is considered to terminate the CFD process.

Figure 1 The plan and the side view of the mesh grid implemented in this study. The red star is the center of the Initial Water Displacement (IWD) located at a distance of 110 km and 50 km in x and y-direction from the origin. The red box contains 40 × 40 grid points, indicating the location for the placement from the IWD mesh grid. The grid resolution is 500 meters in all the modeling area. The depth of the blue cells is 5000 meters. The width of the modeling area is 100 km, and the length of the modeling area is 150 km long. The brown cells indicate depths lesser than 5000 meters. The slope of the seashore is shown in the side view. The arbitrary tide gauge stations are shown by circles nominated by the number stating for the station number. Station 1 is closest to the IWD, and station 4 is the last station that receives the tsunami waves. The shoreline is the vertical line in the left of in the “Plan view”, which passes through Station 4. The stations are intentionally kept in one line to illustrate the effect of bathymetry better.

Let name the IWD with ‘a=1’, ‘b=1’ and ‘c=1’ as the “Mother IWD”. A CFD was modeled using the Mother IWD as the surface of water elevation before propagation, and the arbitrary tide gauges have recorded the tsunami signal and named the generated records by the “Mother IWD” as “Mother Records 1” to 4. The maximum amplitudes of “Mother records” were measured.

The cumulative energy of the Mother Records was also computed using Eq. (3). Where \( x_n \) are the discrete values of the height difference from the sea level which are recorded by the arbitrary tide
gauges. Note that the term “energy” is for characterizing the tide gauge signals and is not the actual measurement of the energy of the signal. The unit of the cumulative energy is (unit of signal)$^2$.

$$E_S = \langle x_n \cdot x_n \rangle = \sum_{-\infty}^{+\infty} |x_n|^2$$  \hspace{1cm} (3)

These bulleted items are used as examples and should be deleted so they do not appear in your document.

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**Figure 1** Effects of the change in scaling and rotation parameters of an initial water displacement caused by a tsunami; The left and the right column shows the effects of the change on amplitude and energy, respectively. The top three rows are results of changes in parameters ‘$a’$, ‘$b$’, and ‘$c$’, while the last row is the results of changes in the rotation parameter ‘$\theta$’. The symbols in the legend are the place where the values are calculated. For each station, a curve is fitted through the values.

**Examples (Optional)**

By having the value and the energy of the ‘Mother Records’, we can compare the effect of scaling and rotation parameters with these results. The three upper rows of Figure 2 show the effect of changes in the scaling parameters ‘$a’, ‘b$, and ‘$c$’, while the last row shows the effect of changes in rotation parameter ‘$\theta$’. Each change in the scaling parameters requires a separate CFD modeling in the region. Therefore, several CFD was attained for the achievement of Figure 2. Increasing the scaling parameter ‘$a$’ increases the ratio of amplitude with the same coefficient of ‘$a$’. However, the ratio of energy is increased with a higher slope and exceeds fifty for ‘$a = 7$’, and above. All of the stations are amplified in amplitude and energy similarly. Increasing the scaling parameter ‘$b$’ increases the ratio of amplitude with slightly higher than the value of coefficient ‘$b$’. However, the ratio of energy increases to a much higher slope and exceeds one hundred for ‘$b = 7$’ and above. For values 1 to 5 of parameter ‘$b$’, all of the stations are amplified in amplitude and energy similarly. Nevertheless, increasing the value of ‘$b$’ Station 4, which is closer to the shore, receives lesser amplification in
amplitude and energy than other stations. Increasing the scaling parameter ‘c’ increases the ratio of amplitude slightly more than any other scaling parameter. The maximum value achieved is 3 in amplitude, and is quite nonlinear for the amplitude amplification ratio, but seems linear for the ratio of energy. The maximum ratio of energy amplification is approximately 10 for ‘c = 10’. Therefore, the ‘c’ parameter receives minimal value in comparison to other scaling parameters. Increasing the rotation parameter ‘θ’ from zero to 90 degrees decreases the ratio of amplitude and energy. After 90 to 180 degrees, the ratio of amplitude and energy is increased again periodically. The periodic change is visible, and two periods are shown in the last row of Figure 2. Figure 2 can be used as a chart to estimate the combined values of scaling and rotation parameters. For example, if we assign any values between 1 to 10 for scaling parameters and 0 to 360 for the rotation parameter, then one could obtain the maximum amplitude and energy of the present IWD by multiplying the value obtained from the Mother IWD for amplitude and energy, with the ratios obtained from Figure 2. In further studies, the work done here can be extended to calculate results for values more than 10.

Conclusions

In this manuscript, we have studied the effect of changing scaling and rotation parameters by using Initial Water Displacements (IWD’s), similar to IWD’s generated by tsunamis. We have proposed an equation for generating the initial water displacement. By testing several values for the scaling and rotation parameters from 1 to 10, we have clearly shown that the scaling parameters ‘a’ (height or z-scale) and ‘b’ (x-scale) better amplify the amplitude and energy of the incoming wave in comparison to the scaling parameter ‘c’ (y-scale) which has the lowest impact on amplification of amplitude and energy. Meaning that if an IWD is extended in the shoreline direction, the impact is much lower when the same IWD is extended perpendicularly to the shoreline direction. The results of this study can be used in the estimation of the maximum amplitude and energy of initial water displacements, by multiplying the ratio of amplitude and energy coefficients to the prior values of mother IWD’s.

References