Introduction

With the development of unconventional shale gas/oil exploration, a common observation is the universal presence of cracks/fractures. The seismic identification and characterization of fractures/cracks are of fundamental importance for searching "sweet spots" in these formations, because in these spots the cracks are usually filled with oil or gas and are aligned due to source rock maturation and stress. Determining the existence and orientation of cracks in a hydrocarbon reservoir is thus an important task, as has been demonstrated for a horizontal-well application. Therefore, a realistic seismic model for the reservoir rocks should be considered.

The commonly used crack-induced seismic-anisotropy models are those of Eshelby-Cheng (Eshelby 1957; Cheng 1993) and Hudson (Hudson 1981). Song and Hu (2014) developed a sphere-equivalency approach to compute the elastic moduli of the cracked solid. Xu et al. (2018) analyzed the effective modulus of isotropic media containing multiple ellipsoidal crack sets with varying orientations using this sphere-equivalency approach. With the new modeling approach, we can model the elastic properties of cracked solid using classic Eshelby theory with much improved accuracy.

However, the above-mentioned theories assume that the background rock medium is isotropic, which may not be suitable for the commonly encountered reservoir rocks, e.g., the case of shale gas or oil reservoirs. This paper extends the sphere-equivalency modeling to a transversely isotropic (TI) background. The key of this approach is equating the far-field elastic waves scattered by the crack in the TI solid to the waves scattered by a spheroid whose elastic moduli are equivalent to those of the same TI solid containing the crack. The modeling results are used as theoretical basis for interpreting experimental and borehole acoustic data, which also provides a validation for the new modeling approach.

Sphere-equivalency theory

Consider the incidence of an elastic wave in an infinite medium containing a crack. The scattered wavefield due to the crack is given by

$$u_i(x) = V \left[ \Delta \rho \omega^2 u_i^0(\zeta) G_{ii}(x,\zeta) - \Delta C_{ijpq} U^{jpqr} \frac{\partial G_{ii}(x,\zeta)}{\partial \zeta_j} \right],$$

(1)

where $u_i^0$ and $e_{ij}^0$ are, respectively, the displacement and strain due to the incident wavefield; $V$ is the volumetric size of the crack; $\Delta \rho$ and $\Delta C_{ijpq}$ are respectively the density and the stiffness-tensor difference between crack and background; $\omega$ is angular frequency; $G_{ii}$ is Green’s tensor. $U$ is the fourth-order strain tensor in the crack alignment coordinates,

$$U = P^{-\delta} T P = P^{-\delta} \left[ I + S \left( C^b \right)^{-\delta} \left( C^c - C^b \right) \right]^{-\delta} P,$$

(2)

where $T$ and $P$ are the strain tensor and coordinate-transformation tensor, respectively; $C^b$ and $C^c$ are the elastic tensor of the background and crack, respectively; $I$ is the identity tensor; $S$ is the Eshelby tensor (Eshelby 1957).

Applying the above formulation to a spheroidal inhomogeneity, designated as the equivalent sphere, results in another scattered wavefield for the sphere. In the sphere-equivalency theory, the elastic moduli of the sphere are taken as the effective moduli of the background medium containing the crack. By making the scattered wavefield due to the crack equal to that due to the equivalent sphere (Xu et al. 2018), the effective elastic stiffness tensor $C^e$ of the cracked medium is derived as

$$C^e = C^c + \phi \left[ I - \phi \left( C^c - C^b \right) S^e \left( C^e \right)^{-\delta} \right]^{-\delta} \left( C^e - C^b \right) U,$$

(3)

where $\phi = V/V^*$ is the volume ratio of the ellipsoidal cracks and the equivalent sphere. For an ellipsoidal crack, $\phi = 4/3 \pi e \alpha^c$, where $e$ and $\alpha^c$ are crack density and aspect ratio, respectively. $S^e$ is the Eshelby tensor (Eshelby 1957) for the spherical inclusion.
Experimental verification

Experimental data measured under controlled laboratory conditions are essential for validating the predictions from theoretical models. In order to study crack-induced seismic anisotropy, de Figueiredo et al. (2018) measured elastic wave velocities and anisotropic parameters in 17 anisotropic laboratory samples at ultrasonic frequency of 500 kHz. The samples were made with a measurable VTI background anisotropy using a layering deposition method. The cracks are parallel to the bedding. Of the 17 samples, one was used as a reference without embedding cracks; the other 16 were embedded with penny-shaped cracks. These cracked samples were divided into four groups, each group consisting of four samples with specific crack aspect ratio (denoted by $\alpha^c$) values ($\alpha^c = 0.08, 0.20, 0.32, 0.52$). For the four groups of samples, the crack density, as calculated for the penny-shaped cracks, was varied in a range from 0 to 0.102.

![Graphs showing experimental data and theoretical predictions](image)

**Figure 1** Measured (dry sample) $P$- (a) and $S$-wave (b) velocities and Thomsen parameters (c) versus crack density for four crack aspect ratio values. The solid and sashed curves are calculated respectively using the sphere-equivalency method and the modified Eshelby-Cheng theory. In general, the sphere-equivalency modeling fits the data better than the modified Eshelby-Cheng theory.
The experimentally measured P- and S-wave velocities are compared with the respective theoretical modeling results in Figure 1a and 1b. The theoretical results from the sphere-equivalency method (solid curve) and the Eshelby-Cheng theory (dashed curve) agree with the experimental results (markers) for low crack densities. With increasing crack density and aspect ratio, the agreement of our theory is significantly better than the Eshelby-Cheng theory. Both theories show poor agreement for the last data set where the aspect ratio is 0.52 and the crack radius is about 3.6mm. This happens because the crack size in this scenario is comparable to wavelength. Therefore, the theory (both the sphere-equivalency and the modified Eshelby-and Cheng modeling) that does not fit this condition is unable to model the data properly.

Figure 1c shows the predicted (solid curve) and measured (markers) Thomsen parameters ($\varepsilon_{Th}^{}$, $\gamma_{Th}^{}$ and $\delta_{Th}^{}$; Thomsen, 1986) versus crack density for the four groups of samples. For comparison, the predictions from the Eshelby-Cheng theory modified for the background VTI are also plotted. In general, the results from the sphere-equivalency modeling agree with the experimental data quite well for all four groups of samples. In comparison, the results from the modified Eshelby-Cheng theory agree with the data mostly in the small aspect ratio and crack density scenarios. The discrepancy between the Eshelby-Cheng theory and the sphere-equivalency modeling was also noted in Xu et al. (2018) for the isotropic background case. The experimental data application demonstrates the validity and accuracy of the sphere-equivalency approach for modeling cracked anisotropic solids.

Field application and results

The field example shown in Figure 2 concerns a shale-gas reservoir from Northwestern China. Vertical fractures were found from visual inspection of the cores. The high gamma ray value of the depth section indicates the shale lithology of the formation, which should have high VTI anisotropy. The Thomsen’s S-wave anisotropy parameter $\gamma_{Th}^{}$ is shown in panel 2 of Figure 2a, which is estimated using the Xu et al. (2017) method. The S-wave anisotropy exhibits significant variations for this massive shale formation, showing large reduction in the intervals of 2720-2735m and 2753-2775m, respectively. The cause of the reduction is again linked to the vertical fractures in these intervals, which can be demonstrated by the fracture-induced azimuthal shear-wave anisotropy results shown in panels 3-5. This anisotropy is obtained by processing the four-component dipole acoustic data (Tang & Cheng 2004) of the depth section. Panel 3 shows the anisotropy (shaded curve). Panel 4 shows the fast (blue) and slow (black) shear waves (i.e., the quasi-SH and quasi-SV waves along the vertical axis), with a significant splitting of the waves corresponding to substantial anisotropy values. The anisotropy map in panel 5, displayed as an intensity image in the 0-360° range, shows that the azimuthal anisotropy is aligned in the NS direction, which, for the fracture-induced anisotropy, is the fracture strike direction. Notice that the magnitude of the azimuthal anisotropy is well correlated with the reduction of the TI anisotropy in the intervals of 2720-2735m and 2753-2775m.

To further analyze the anisotropy characteristics, we theoretically model the formation anisotropy variation by adding vertical cracks with various crack density (incremented by 0.01) and crack aspect ratio ($ac =0.01, 0.1, 0.2 \ldots 0.9$) values. Figure 2b shows the crossplot of S-wave TI anisotropy versus horizontally propagating S-wave velocity for the varying crack parameters. The data (markers) from Figure 2a, color coded with the azimuthal anisotropy value, are also plotted. Both the modeling and the data show the general trend of decreasing TI anisotropy with increasing (vertical) crack density. Figure 2c shows the crossplot of S-wave azimuthal anisotropy versus vertically propagating S-wave velocity for the varying crack parameters, together with the data (markers, color coded with azimuthal anisotropy values) of Figure 2a. Both the modeling and data show the general trend of increasing azimuthal anisotropy with increasing crack density. The theoretical modeling suggests that the variation characteristics of the measured acoustic anisotropy data are caused by the vertical fractures in this depth section. Delineation of the fracture system can indicate sweet spots for hydrocarbon production in the shale formation.
Figure 2: Theoretical modeling and interpretation of the borehole acoustic anisotropy measurements (a) for a shale formation. Note that the S-wave TI anisotropy (panel 2) and the azimuthal anisotropy (panels 3, 4 and 5) are anti-correlated. The theoretical modeling results (black squared lines in (b) and (c)) show that the measured anisotropy (markers) characteristics can be explained as due to the presence of vertical fractures in this depth section.

Conclusions

We have developed a theoretical analysis for modeling the elastic anisotropy of cracked anisotropic media using the method of sphere-equivalency of effective scattering. The validity and accuracy of the new modeling method have been verified by the anisotropy data from a controlled laboratory experiment. The analysis and processing of field logging data further verifies the reliability and applicability of the proposed method. In general, the new modeling analysis can be used for interpreting seismic anisotropy measurements from cracked anisotropic crustal rocks.

References