Deep electrical structure of Cascadia derived from 3D anisotropic inversion of USArray long-period magnetotelluric data

Introduction

For a long time, geologists believed that the frequent occurrence of earthquakes in the west coast of the United States is closely related to the existence of a north-south Cascadia subduction zone in the deep part of the region. A large amount of geophysical exploration work has been carried out, and the National Science Foundation EarthScope/USArray program has collected a large amount of long-period magnetotelluric (MT) data in the area. Many geophysicists have studied these data. Through data analysis and three-dimensional (3D) MT inversion, the electrical resistivity distribution of the crust, oceanic crust, and even the upper mantle of the earth has been obtained. From the published results, the approximate distribution of the Cascadia subduction zone in the area can be inferred, but they also pointed out that there is significant electrical anisotropy in the deep underground of this area (Patro and Egbert, 2008; Meqbel et al., 2014). The most common data interpretation methods in MT are either based on the isotropic model (Siripunvaraporn et al., 2009; Jahandari and Farquharson, 2017), or based on 1D and 2D anisotropic models (Baba, 2006; Chen, 2012), which cannot meet the interpretation accuracy of the 3D distribution of underground electrical structures in the exploration area. Cao and Yin et al. (2019) studied the 3D MT anisotropic inversions, but only for three principal conductivities, without considering the dip angle of the anisotropy. More and more data analysis reveals the anisotropic angle can effectively influence the MT signal. This brings new challenges to 3D MT inversion and interpretation.

To solve the problem with electrical anisotropy in 3D MT inversions, we propose a 3D MT anisotropic inversion method that considers simultaneously all principal conductivities and rotation angles. After testing on synthetic data (not shown in the abstract), we applied the method to the Long-period MT data collected from the west coast of the United States.

Method

Inversion strategy

For our 3D MT anisotropic inversions, we construct a regularized objective functional that includes a data misfit term, and two Tikhonov regularization terms, i.e.

$$\psi = \psi_d + \lambda_w \psi_w + \lambda_e \psi_e,$$

where $\psi_d$ is the data misfit. Considering the Gaussian characteristic of the error, we use the L2-norm to construct the data misfit and write it as

$$\psi_d = |d^{obs} - f(m)|^2 C_d^{-1} |d^{obs} - f(m)|,$$

where $d^{obs}$ denotes the observed data, $f(m)$ denotes the predicted data calculated via the forward modeling of the model $m$, $C_d$ denotes to the covariance matrix of the data. Since the impedance tensor is more sensitive to electrical anisotropy than other MT responses, we use it all components in the inversion, i.e.

$$Z = \begin{bmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{bmatrix}.$$  (3)

In order to predict the tensor impedance responses for the inverted model, we use the unstructured finite-element method to solve the double curl equation for the electric field in anisotropic media (Jin, 2002; Cao et al., 2019). For that purpose, we discretize the model space with unstructured tetrahedral elements. According to the Galerkin method, the electric field in each element satisfies the following equation:

$$\sum_{j=1}^{6} \int_{\Omega} (\nabla \times \mathbf{N}_j) \cdot \left( \nabla \times \mathbf{E}_i \right) d\Omega + i \omega \mu_0 \sum_{j=1}^{6} \int_{\Omega} \nabla \left( \mathbf{N}_j \sigma \mathbf{N}_j \right) \cdot \mathbf{E}_i d\Omega = 0 \quad j = 1, \ldots, 6,$$  (4)
where \( \sigma = [\sigma_x \sigma_y \sigma_z \alpha \beta \gamma]^T \) is the conductivity tensor that has six independent parameters. \( m = [m^1, m^2, m^3, m^4, m^5, m^6]^T \). After Yin (2000), the conductivity tensor \( \sigma \) can be obtained by the following Euler rotations:

\[
\sigma = R_x(\alpha)R_y(\beta)R_z(\gamma)\left[ \begin{array}{ccc}
\sigma_x & 0 & 0 \\
0 & \sigma_y & 0 \\
0 & 0 & \sigma_z
\end{array} \right] R^T_y(\gamma) R^T_z(\beta) R^T_x(\alpha),
\]

(5)

where \( \alpha, \beta, \) and \( \gamma \) are the rotation angles. \( R_x, R_y, \) and \( R_z \) are matrices for rotations respectively around the three principal axes. Assembling all elements together and applying Dirichlet conditions to the outside boundary, we obtain the following linear equations system:

\[
\textbf{KE} = \textbf{b}.
\]

(6)

Solving Eq.(6) for two source polarizations (TE and TM), we can obtain the electric field. Then, by using the numerical interpolation, we can obtain the full impedance tensor.

In order to penalize excessive structures, we impose a structure regularization term \( \psi_m \) to control model roughness. To adapt to unstructured grids, we use the following weighted sum to define the roughness:

\[
\psi_m = ||s||_2 = \sum_{i=1}^{M} \sum_{j=1}^{N} W_{ij} \left( \frac{\Delta m_{ij}}{d_{ij}} \right)^2,
\]

(7)

\( W_{ij} \) denotes the volume weights between the current unit and its adjacent units, \( d_{ij} \) is the centroid distance between the units.

To distinguish the conductivity between the isotropic and anisotropic media, we impose additional Tikhonov regularization term \( \psi_e \) (Pain et al., 2003) in the objective functional that is defined as

\[
\psi_e = \sum_{i=1}^{M} \left( m^i_1 - m^i_m \right)^2 + \left( m^i_1 - m^i_m \right)^2 + \left( m^i_1 - m^i_m \right)^2.
\]

(8)

The weight factor \( \lambda_e \) of the additional Tikhonov regularization depends on the anisotropic characteristics of the medium. We use the depth weighting method to define it \( \lambda_e(z) \).

To minimize the objective functional, we need to calculate the gradient \( \nabla \psi \), namely

\[
\nabla \psi = -2\nabla \psi_e \text{grad} \psi + \lambda_e \nabla \psi_m + \lambda_e \nabla \psi_e,
\]

(9)

where \( J_e = \frac{\partial \textbf{K}}{\partial m} \) is the sensitivity matrix that can be expressed as

\[
J_e = \frac{\partial \textbf{K}}{\partial m} = \lambda_e \frac{e_{tort}}{\text{Vol}} \int N \left( \frac{\partial \sigma}{\partial m} \right) N dV.
\]

(10)

With the gradient, we can obtain the model updates by using the L-BFGS algorithm (Liu and Nocedal, 1989), i.e.

\[
\Delta m = -\nabla \psi \text{grad} \psi,
\]

(12)

\[
D = \left( I - \rho \frac{s}{\|s\|} \right) \text{grad} \psi + \rho \frac{s}{\|s\|} \text{grad} \psi,
\]

(13)

where \( s \) is model displacement, \( \text{grad} \psi \) is gradient change and \( \rho = 1/(\|s\|^2) \). \( \nabla \psi \) is obtained by the method of linear search. The termination of the inversion process depends on the data fitting. We use the root mean square (RMS) of data misfit to judge whether the inversion has converged, namely

\[
\text{RMS} = \sqrt{\frac{1}{2N} \sum_{i=1}^{N} \left[ \frac{d_i^{\text{obs}} - d_i^{\text{pred}}} {\|y_i\|} \right]^2}
\]

(14)

Examples
We use the long-period (100-100000s) MT data collected by USArray in our 3D anisotropic inversion. There are a total of 109 measuring sites, and the average site spacing is about 60km, the schematic diagram of the study area is shown in Figure 1. This survey site covers Washington and Oregon on the west coast of the United States, which is the area where the Cascadia subduction zone is located. The data have been processed by a standard robust remote reference approach, and 5% Gaussian noise is estimated for the impedance tensor. In the following, we will use resistivity to replace the conductivity in our inversion.

The model domain has a dimension of 10000 × 10000 × 10000km consisting of 565,870 tetrahedral cells. Our inversion domain has a dimension of 1000km×1000km×200km. The Pacific Ocean has already been determined according to the coastline, with a depth of around 5km and a conductivity of 0.3ohm-m. The initial model is assumed to be a half space of 100 ohm-m (except for the ocean).

Figure 2 shows the results from our 3D MT inversion for isotropic and anisotropic models. For comparison, we choose the most robust isotropic inversion results (with 122 iterations and a $RMS = 2.46$), and 3D anisotropic inversion results (with $RMS = 2.47$). In 3D MT anisotropic inversion, we find that there are three parameters that are insensitive. Thus, we don’t display them in the Figure 2.

![Schematic diagram of the study area](modified from Patro and Egbert,2008)

![3D views of the inversion results](a) Isotropic inversion; (b) resistivity-x of anisotropic inversion; (c) resistivity-y of anisotropic inversion;(d) rotation angle $\gamma$. All resistivities in ohm-m.
Conclusions

We have successfully developed a 3D MT inversion algorithm for anisotropic earth models. The numerical experiments have proved the effectiveness of our inversion method. The anisotropic characteristics in the Cascadia subduction zone are identified from our inversions. Hope that this will help further analyze the earthquake-prone zone.

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References

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