**Introduction**

Full waveform inversion (FWI) can produce the high-resolution subsurface medium parameter models (Tarantola, 1984; Plessix, 2006). While it has a low efficiency, especially in the elastic case (Tarantola, 1986; Brossier et al., 2009; Plessix and Pérez Solano, 2015; Zhang et al., 2016). In order to improve the efficiency of FWI, the parallel algorithm (Shin et al., 2014; Zhang et al., 2014) is usually adopted based on the high-performance computer device but does not essentially reduce the computational amount. In order to reduce the total computation volume, the compressive sensing or random data decimation methods (Li et al., 2012) only choosing some of the sources to perform the inversion can reduce the computation cost to some degree. However, insufficient source sampling would degrade the inversion results (Zhang et al., 2018).

Another crucial strategy to reduce the computational cost is the simultaneous-source FWI (SSFWI) algorithms (Krebs et al., 2009; Schuster et al., 2011). Nevertheless, conventional SSFWI algorithms face the problems of crosstalk artifacts and fixed-spread acquisition setup, which both result in the impracticability of these algorithms. Different from the conventional strategies to solve the above two bottlenecks, Huang and Schuster (2018), Zhang et al. (2018) proposed a novel crosstalk-free SSFWI algorithm with frequency-selection method (Dai et al., 2013). The blended wavefields can be deblended so that there are no crosstalk artifacts and fixed-spread assumption. Unfortunately, the frequency-selection algorithm, similar to the frequency-domain FWI using only limit frequencies to perform the inversion, is sensitive to the noise in the observed data. Besides, the convergence rate of this algorithm is very slow or a large number of iterations are required due to the random frequency selection strategy (Dai et al., 2013; Huang and Schuster, 2018; Zhang et al., 2018) is used.

In this paper, we propose a regularized crosstalk-free SSFWI algorithm to improve the inversion convergence rate and the robustness to background noise. The crosstalk-free frequency-selection source-encoding algorithm is adopted with the singular value decomposition (SVD) truncation to denoise the parameter gradient. A comparison to the shaping regularization (Xue et al., 2017) is conducted to verify the feasibility and advantage of this algorithm.

**Method**

Different from the conventional time-domain wavefield modeling algorithm, the harmonic wavelets with different angular frequencies $\omega_{ss}$ are used as the encoding operator to perform the simultaneous-source simulation, which can be expressed as

$$F(x, t) = \sum_{ss=1}^{Es} A^F_{ss}(x_{ss}) \sin \left[ \omega_{ss} t + \theta^F_{ss}(x_{ss}) \right],$$

where $F$ denotes the super shot wavelet, $A$ and $\theta$ represent the amplitude and phase information, $ss$, $Es$ and $x_{ss}$ are the index, number and coordinate of simultaneous sources in one super shot.

In order to deblend the simultaneous-source wavefields to avoid the crosstalk artifacts, we define the reference signals with the corresponding angular frequency $\omega_k$, $k \in \{1, 2, ..., Es\}$.

$$f^r = \sin(\omega_k t), \quad f^R = \sin(\omega_k t + \pi/2).$$

Based on the orthogonality of trigonometric functions, we can define and calculate the intermediate variables for the $k$th source during the wavefield simulation

$$X_k = \frac{1}{T_l} \int_{T_m-T_l}^{T_m} \phi f^r dt = \frac{1}{2} A^r_k \cos(\theta^r_k), \quad Y_k = \frac{1}{T_l} \int_{T_m-T_l}^{T_m} \phi f^R dt = \frac{1}{2} A^r_k \sin(\theta^r_k),$$

where $T_m$ denotes the maximum simulation time, $T_l$ represents the least common multiple period of all angular frequencies of harmonic wavelets in one super shot, $\phi$ is the forward-propagated simultaneous-source wavefield. Consequently, we can recover the amplitude and phase fields of the $k$th source from...
the blending wavefield

$$A_k^\phi = 2\sqrt{X_k^2 + Y_k^2}, \quad \theta_k^\phi = \tan^{-1}(Y_k/X_k). \quad (4)$$

The same processing can be applied to the backward-propagated blended adjoint wavefields. Based on eq. 4, the wavefield can be expressed as a few ($E_x$) frequency-domain snapshots, which avoids the great memory requirement to a large degree. More importantly, the forward- and backward-propagated simultaneous-source wavefield deblendings avoid the influence of simultaneous-source crosstalk artifacts and data mismatch problem caused by the fixed-spread assumption.

The aforementioned algorithm uses harmonic wavelets, similar to the frequency selection in the frequency-domain FWI, so that it is sensitive to the background noise in the seismic data. To improve the inversion quality and convergence rate of the algorithm, we adopt the SVD truncation to denoise the parameter gradient. Performing SVD to the model gradient, we can obtain

$$g = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \quad (5)$$

where $g$ denotes the gradient. In order to suppress the artifacts in the gradient caused by the background noise in observed data, we can truncate the small singular values in eq. 5

$$\hat{g} = U_1 \Sigma_1 V_1. \quad (6)$$

Based on eq. 6, the convergence rate and robustness of the algorithm can be improved dramatically.

**Examples**

We use the Overthrust model to conduct the tests. The true velocity (Fig.1(a)) and a full-band Ricker wavelet with dominant frequency of 15.0 Hz are used to generate the unblended mimic observed seismic data. Fig.1(b) shows the initial velocity model for SSFWI. Fig.2 shows the noise-free SSFWI results with 20, 33, 33, 50 and 100 sources per super shot for 5 frequency bands. It can be seen that although tens or even hundreds of sources are encoded as one super shot, the inversion results can still recover the subsurface velocity parameter perfectly. Compared to the result without regularization, the regularized result with SVD truncation has a higher resolution (dotted box areas), especially for the small velocity anomalies in the right box. Fig.3 denotes the convergence rates of 5 frequency bands without and with the SVD truncation, from which it can be seen that the regularized convergence rate is faster than that without preprocessing.

![Figure 1](image1.png)

**Figure 1** The (a) true and (b) initial velocity models.

![Figure 2](image2.png)

**Figure 2** Inversion results of noise-free data (a) without and (b) with SVD truncation.

To verify the robustness of the proposed algorithm, we add random noise into the observed seismic data with SNR=10.0dB. Besides, we also adopt the shaping regularization to make a comparison to SVD truncation, the corresponding results are showing in Fig.4. Due to the background noise in observed data, the crosstalk-free SSFWI result becomes blurring (Fig.4(a)). The convergence rate is extremely
**Figure 3** Convergence rate of noise-free FWI (a) without and (b) with SVD truncation. There are 5 frequency bands and each band conducts 15 iterations.

slow and even nonconvergent. After adding the shaping regularization (Fig.4(b)), the artifacts caused by the background noise in data are suppressed to a large degree but the velocity boundaries are smoothed. Besides, in this case, the shaping regularization can only improve the convergence rate to a little degree (Fig.5(b)). In contrast, our algorithm (Fig.4(c)) can not only dramatically suppress the artifacts but also protect the velocity boundaries commendably. Moreover, as shown in Fig.5(c), the convergence rate of our algorithm can be improved observably.

**Figure 4** Inversion results of noisy data (a) without regularization, with (b) the shaping regularization and (c) SVD truncation.

**Figure 5** Convergence rates of noisy FWI (a) without regularization, with (b) the shaping regularization and (c) SVD truncation.
Conclusions

We propose the regularized crosstalk-free SSFWI algorithm with SVD truncation. Based on this algorithm, the blended simultaneous-source wavefield can be decomposed, avoiding the crosstalk artifacts and fixed-spread acquisition assumption. SVD truncation are applied to preprocess the model gradient. Synthetic seismic data examples verify that our algorithm can not only accelerate the convergence rate but also improve the inversion quality dramatically, especially in the noisy case. Compared to the shaping regularization, our algorithm can protect the velocity boundaries with a high resolution.

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References


