Huber inversion-based reverse-time migration with de-primary imaging condition and sparse constraint in curvelet domain

Introduction

Least-squares reverse-time migration (LSRTM) (Dai and Schuster, 2013) formulates reverse-time migration (RTM) (Zhang et al., 2011) in the least-squares inversion framework to produce the optimal reflectivity image. LSRTM can produce images with more balanced amplitudes, higher resolution, and fewer artefacts caused by missing data, truncation, noise and operator mismatch than conventional RTM thanks to the inverse operator applied by LSRTM. However, conventional LSRTM utilizes L2-norm as the objective function. Consequently, the inversion is dominated by strong seismic events or other outliers in data. For instance, the events originating from the boundary of salt domes is much stronger than the events from the interfaces of sedimentary rock. These strong events will dominate the whole inverse process of LSRTM, resulting in an inadequate update to the region under the salt domes. The second weakness of conventional LSRTM is that it still uses the zero-lag cross-correlation imaging condition, which generates the low-wavenumber migration artefacts, especially above strong reflectors, due to two-way wave equations employed (Liu et al., 2011). In addition, the zero-lag cross-correlation imaging condition also generates false high-wavenumber reflectors in geological complex area. Thirdly, the inversion process of conventional LSRTM also produces high-wavenumber reflectors to overfit the record because of the ill-conditioning nature of geophysics problems as well as the modelling engine that only simulates the record partially.

In this paper, we propose a new strategy to achieve LSRTM for solving the problems illustrated above. Firstly, we employ a Huber-norm (Huber, 1964) as the objective function of the inversion. Thus, the inversion acts as L1-norm inversion for large residual but L2-norm inversion for small residual. Consequently, it is strongly convex when close to the minimum and less steep for extreme values, mitigating the problem that large residual dominates inversion in LSRTM. Since the inversion is not based on L2-norm only, we call it Huber inversion-based RTM. Secondly, the de-primary imaging condition (Fei et al., 2015) is employed to formulate the gradient instead of the zero-lag cross-correlation, which causes both low- and high-wavenumber artefacts. Finally, we use the L1-norm sparse constraint in the curvelet domain as the regularization term in the objective function to remove high-wavenumber noise. This is because the curvelet transform can sparsely represent images at multiple scales and angles, but also has a stronger ability to express edge information than other transforms, such as wavelet transform (Candes and Guo, 2002). As the proposed Huber inverse-based RTM is no longer a quadratic problem, we utilized a preconditioned nonlinear conjugate-gradient method (PNCG) and the improved iterative soft thresholding (IST) method to solve this nonlinear sparse constrained inversion problem effectively. The numerical example of the Sigsbee2A model demonstrates that our proposed method can improve the image quality and show more structural details in poorly illuminated areas.

Theory

The objective function of Huber inversion-based RTM can be expressed as
\[
\phi_d = \delta^2 \left( \frac{1}{1+(d-d_{\text{obs}})/\delta} \right)^2 - 1, \tag{1}
\]
where \(d\) denotes predicted data, \(d_{\text{obs}}\) represents the observed data, and \(\delta\) is the threshold of Huber-norm. When the data error \(d-d_{\text{obs}}\) is greater than \(\delta\), Huber-norm is equivalent to L1-norm. Otherwise, it is equivalent to L2-norm.

To obtain the optimal reflectivity model by minimising the objective function, the mapping relationship between predicted data and reflectivity is essential. Generally, there are two simulation methods for predicted data: one is based on Born approximation, and the other is based on Kirchhoff approximation. In this paper, we used reflected data simulated by the Kirchhoff approximation (Xu et al., 2011):
\[
d(x, \omega) = \int_{\Omega} G_r(x, |x,\omega|) m(x) G_{e}(x | x,\omega) x(\omega) dx, \tag{2}
\]
where \( s \) is the source wavelet, \( m(x) \) represents the stacked reflectivity image, \( G_s(x|x,\omega) \) is Green's function from the source point \( x \), to the reflection point \( x \), \( G_s(x|x,\omega) \) is Green's function from the reflection point \( x \) to the receiver point \( x \), and \( i\omega \) gives a 90° phase shift to the simulated data, which is essential for zero-phase imaging.

Substituting equation (2) into equation (1) can obtain the gradient of the objective function

\[
\frac{\partial \phi_{\mu}}{\partial m(x)} = \left( \frac{\partial d}{\partial m(x)} \right) - \frac{d(x) - d_{obs}(x)}{\sqrt{1 + \left( \frac{d(x) - d_{obs}(x)}{\delta} \right)^2}},
\]

where \( G_s(x|x,\omega) s(x,\omega) \) represents the forward wavefield that is generated by propagating the seismic source from the source point \( x \) to the reflection point \( x \), and \( G_r(x|x,\omega) (d - d_{\text{obs}}) \) represents the backward wavefield that is generated by propagating the residual from the reflection point \( x \) to the receiving point \( x \).

To further improve the quality of migration imaging by suppressing high-wavenumber noise, we add L1-norm sparsity constraint in the curvelet domain as the regularization term to the objective function

\[
\phi = \phi_{\mu} + \lambda C^{-1}\|m\|_1,
\]

where \( \lambda \) is the trade-off parameter between the data misfit term and the regularization term, \( m \) represents the stacked reflectivity model, \( C \) and \( C^{-1} \) represent the forward and inverse curvelet transform of the model, respectively. \( \| \|_1 \) represents L1-norm. Herein, we used the IST method to minimise functional (4). IST is a special case of the proximal method, which solves a general optimization problem,

\[
\min g(m) + h(m),
\]

iteratively with

\[
m' = \text{prox}_{\alpha h}(m_{i-1} - \alpha \nabla g(m_{i-1})),
\]

where \( g(m) \) is a smooth function, e.g. \( \phi_{\mu} \), \( h(m) \) is a convex function but not differentiable everywhere, e.g. \( \lambda C^{-1}\|m\|_1 \), \( \alpha \) is the step-length at the \( i \)-th iteration, and \( \text{prox}(\cdot) \) is called as the proximity operator and is the soft thresholding operator for IST. Originally, the proximal method operates along the gradient direction, i.e. \( \frac{\partial \phi_{\mu}}{\partial m(x)} \). We found that the Polak-Ribière conjugate-gradient direction preconditioned by the stack of all shots' source wavefield energy can be used in IST for faster convergence. We then have an improved IST method described in Algorithm 1. Generally, the optimal \( \lambda \) is found by trial and error.

**Algorithm 1** The preconditioned nonlinear conjugate-gradient method with Polak-Ribière for Huber inversion-based RTM.

- **Input:** initial \( m \), \( \lambda \), \( N \);
- \( n = 0, r = \frac{\partial \phi_{\mu}}{\partial m} \);
- Compute the preconditioner \( M \);
- \( s = M^{-1} r, \quad p = s, \quad \delta_{\text{res}} = r' p \);
- While \( n < n_{\text{max}} \) do
  - \( \alpha = \arg\min_{\delta m} \phi_{\mu}(m + \alpha \delta m) \), where \( \delta m \) is a small perturbation along \( p \) and \( \alpha \) is the step length obtained through line search.
  - \( \hat{m} = C(m + \alpha \delta m) \)
- **Output:** \( \hat{m} \) as the final solution.
\[ m = C^{-1}(T_s(\hat{m})), \text{ where } T_s(\hat{m}) = \text{sgn}(\hat{m}) \max(0,|\hat{m}| - \lambda) \]

\[ r = \frac{\partial \phi_{\delta_{\text{ew}}}}{\partial m} \]

\[ \delta_{\text{old}} = \delta_{\text{ew}}, \quad \delta_{\text{ew}} = r's, \quad s = M^{-1}r, \quad \delta_{\text{ew}} = r's \]

\[ \beta = \max\{(\delta_{\text{ew}} - \delta_{\text{old}})/\delta_{\text{old}},0\}, \quad p = s + \beta p \]

\[ n = n + 1 \]

**Numerical Example**

In this section, we test the proposed method on a synthetic data set from the Sigsbee2A model. The seismic data set is generated by using the fine stratigraphic velocity model (figure 1a). The data set includes 160 shots, which are excited with a shot spacing of 137.16 m and a receiver spacing of 45.72 m. The minimum and maximum offsets are 0 m and 7943.85 m, respectively. The source wavelet is a Ricker wavelet with a peak frequency of 12 Hz. The total recording duration is 12 s with a sampling interval of 1 ms. The sources and receivers are placed at 76.2 m below the surface. A smoothed velocity model shown in figure 1b is used for migration.

Figures 2a and 2b show the RTM images with the zero-lag cross-correlation imaging condition and the de-primary imaging condition, respectively. As can be seen, strong low-wavenumber migration artefacts in figure 2a affect the imaging result severely: it covers up the real high-wavenumber reflectors. The de-primary imaging condition suppressed the low-wavenumber (in fact as well as high-wavenumber) migration artefacts effectively. But subsalt reflectors are still unclear due to poor illumination. Figures 3a and 3b depict the results of LSRTM with the zero-lag cross-correlation imaging condition and de-primary imaging condition, respectively. As can be seen, LSRTM with the de-primary imaging condition produces the image with higher resolution and balanced amplitudes. However, the area pointed by the blue arrow is still not imaged. Therefore, we tested Huber inversion-based RTM with the de-primary condition. The continuity of the stratum (indicated by the blue arrow in figure 3c) is improved. This demonstrates that Huber inversion-based RTM with the de-primary condition has the capability to image complex models even with insufficient illumination. However, the inversion process also generates high-wavenumber migration artefacts while promoting the resolution of migration images. To suppress those noise, we added a sparse constraint in the curvelet domain to the inversion. Figure 3d shows the result, in which the high-wavenumber migration artefacts are suppressed largely.

![Figure 1](image1.png)  
**Figure 1** (a) The Sigsbee stratigraphic velocity model. (b) The migration velocity model.

![Figure 2](image2.png)  
**Figure 2** (a) The result of RTM with (a) zero-lag cross-correlation imaging condition and (b) the de-primary imaging condition.
Conclusions

Standard LSRTM can produce better images than conventional RTM. However, three problems still exist: (1) inversion can be dominated by strong events in the residual; (2) low-wavenumber artefacts in the gradient affects convergence speed and imaging results; (3) High-wavenumber noise is also enhanced as iteration increase. To solve these problems, we used Huber-norm as the objective function, the de-primary imaging condition and L1-norm of reflectivity in the curvelet domain as constraint conditions for Huber inversion-based RTM. To speed up convergence, we used PNCG and the improved IST method to effectively solve the nonlinear problem. The numerical example on the Sigsbee2A model shows that the proposed method has greatly improved the migration image, especially the subsalt image, compared with traditional RTM and LSRTM.

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References