Introduction

Velocity analysis constitutes the step of the seismic reflection processing aimed at estimating a velocity field of the P-waves. The knowledge of a velocity model, even if approximate, is necessary to perform subsequent conventional operations such as geometrical spreading and normal move-out corrections, as well as advanced procedures such as migration and other inversions.

Velocity analysis is accomplished by measuring the lateral coherency of the data samples along hyperbolic trial trajectories. The most commonly used method for the evaluation of coherency is the Semblance functional (Neidell and Taner, 1971), which is generally robust against random noise and not time-consuming. However, it produces low-resolution coherencies and it is not adequate if the data are affected by coherent noise or in case of events close in time and/or velocity.

Nowadays, different algorithms have been developed which are able to provide coherency measures with a high-resolution both in time and velocity (Sguazzero and Vesnaver, 1987; Key and Smithson, 1990; Spagnolini et al., 1993; Sacchi, 1998; Abbad and Ursin, 2012; Tognarelli et al., 2013; Gong et al., 2016; among others). All these methods assume a stationary nature of the recorded signals and, for this reason, they adopt a constant set of parameters while computing the velocity spectra.

The panels estimated in this way can generally be considered adequate for the location, in the time-velocity plane, of the most important reflected events. However, if the analysis is performed on data recorded in complex geological contexts characterized by low signal-to-noise ratio and strong lateral and vertical attenuation phenomena, a fixed parameter setting is no longer suitable for the computation of velocity spectra.

This work presents a new coherency functional for the velocity analysis of seismic reflection data. The functional performs the coherency measure in the time-scale domain computed by the multi-resolution technique of Continuous Wavelet Transform (CWT) (Daubechies, 1990) providing a three-component analysis: time, velocity and scale (or frequency). By taking advantage of the multi-resolution CWT approach, the functional presented can perform coherency measures robust against noise and, at the same time, it provides 3D velocity spectra with high-resolution over time, velocity and scale (or frequency). As a result of this, in case of data characterized by the occurrence of non-stationary signals, the functional is particularly useful since it is able to explore the time-velocity variations also over the frequencies (scales).

The first part of this work describes the wavelet-based coherency functional; the second part shows an application to a synthetic case and to a field data-set characterized by very low signal-to-noise ratio, strong multiple contamination and weak sub-basalt primary reflections strongly affected by attenuation. In both examples, the results are compared with those carried out by traditional Semblance and by the unconventional high-resolution functional of Complex-Matched Semblance (Spagnolini et al., 1993; Tognarelli et al., 2013).

Method

In the 2D form, i.e. considering a fixed scale, the wavelet-based coherency functional is defined as:

\[
C_{WS} = \frac{1}{M} \sum_{i=0}^{\frac{M}{2}} \sum_{j=0}^{\frac{M}{2}} \left| \left( \sum_{t=0}^{\frac{T}{2}} W_j(t, v_{stack}) \right)^2 \right|
\]

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\]

(1)

where \( W \) is a common scale (or common frequency) section computed by the Continuous Wavelet Transform of a common midpoint gather (CMP). Index \( j \) refers to the \( M \) trace and \( T \) is the width of the time window. The coherency measure \( C_{WS} \) is then repeated for all the considered scales giving a 3D space defined by: time, velocity and scale (frequency).

Equation 1 is similar to the semblance coefficient formula. Here, the energy evaluated along the trial hyperbolic trajectory normalized to the total energy estimated along the same trajectory is computed in the time-scale domain.

In the Results section, the spectra computed by the new wavelet-based functional, called Wavelet Semblance, will be compared with those performed by the standard Semblance and by the
unconventional Complex-Matched Semblance (Spagnolini et al. 1993; Tognarelli et al., 2013, 2016), which uses an estimated or known analytic wavelet to filter the data before computing the coherency measure. It is defined as:

\[
C_{CM} = \frac{1}{M} \sum_{t=t_0-T/2}^{t=t_0+T/2} \sum_{i=1}^{M} \left( D_i; t_0, v_{stack} \right) \]

where \( D \) are the analytic data filtered by the analytic wavelet.

**Results**

The first example refers to synthetic data. Figure 1 shows the depth model used to generate the simulated CMP by using the reflectivity method. The model consists of six interfaces separating a top water-layer, a sedimentary layer (S1), a thick (ca 600-m) basaltic bed, followed by three sedimentary strata (S2, S3, S4), for a total model depth of ca 4500-m. This model is designed to cause an overlap at a near-zero offset of the top basalt reflection with the first sea-bottom multiple at about 3-s.

The high attenuation value (\( \alpha=0.91 \)) ascribed to the basalt layer produces an important effect on the sub-basalt reflections. Data are generated by propagating a 20-Hz central frequency wavelet and by using a 2-ms sample rate. Figure 2 shows the simulated seismogram (on the left) with added Gaussian noise. The black arrows indicate the time location of the simulated sub-basalt reflections, which are strongly affected by attenuation and mainly buried in the noise.

Figure 2 compares the velocity spectra computed by the Semblance functional, by the Complex-Matched Semblance (CM) where a Ricker wavelet with central frequency of 15-Hz is used as matching filter, and by the Wavelet Semblance (WS). Only the scales

![Figure 1](image1.png)

**Figure 1** Depth model (from Tognarelli et al., 2013) used to compute the synthetic CMP gather.

![Figure 2](image2.png)

**Figure 2** Left: synthetic seismogram with added noise. Left to right: velocity spectra computed by the Semblance, by the Complex-Matched Semblance using a 15-Hz matching filter, and by the Wavelet Semblance functional (only the scales corresponding to 20, 15 and 10-Hz are illustrated).
corresponding to 10-Hz, 15-Hz and 20-Hz are shown. As expected, the experiment demonstrates that the Semblance is unable to detect the sub-basalt reflections. It produces weak coherencies that cannot be considered reliable.

The CM functional as well as the WS functional locate the sub-basalt reflections with a high degree of accuracy. The key point is that CM uses a fixed parameter setting for the analysis (i.e. the same matching filter) and that it also requires the definition of an estimated or known wavelet. Such wavelet is unsuitable when the signals are non-stationary, as in the discussed case. Instead, WS does not require any a-priori information and it naturally explores the coherencies along different common frequency panels. In this way it is able to detect the time-velocity-frequency occurrence of the weak sub-basalt reflections. The CM and the WS spectrum at 15-Hz (Figure 2) shows that the latter is characterized by higher resolution and more energetic coherencies.

Furthermore, the wavelet transform produces efficient filtering (in this example I have employed a complex Morlet mother wavelet) that can be compared to the filtering produced by the Ricker wavelet, thus assuming a perfect source wavelet estimate. In addition, the WS spectra of Figure 2 show that, if is considered a lower frequency (i.e. 10-Hz), more energetic coherencies of the sub-basalt reflections are computed.

The second example (Figure 3) discusses the application to a field data case pertaining to an offshore acquisition. The sea-bottom is located at approximately 1.5-s and the top basalt reflection is around 3-s. As in the synthetic example, the seismogram of Figure 3 shows no evident reflections below the top basalt resulting from the contamination of multiples and noise. Figure 3 shows that the velocity spectrum computed by the Semblance is unable to detect the sub-basalt primaries. By adopting a 15-Hz Ricker wavelet as matching filter, the CM identifies some possible weak sub-basalt reflections highlighted by the occurrence of coherencies below 3-s in the velocity range between 2 and 3-km/s.

Again, WS evidences the capability to explore the trial time-velocities over trial scales with no necessary a-priori information. In the specific case, WS spectra show that 20-Hz are inadequate to produce an exhaustive velocity panel; 15-Hz are suitable for the detection of the reflections above the basalt (3-s); the sub-basalt reflections can be detected by considering common scales corresponding to 10-5-Hz.

**Figure 3** Left: real CMP gather. Left to right: velocity spectra computed by the Semblance, by the Complex Matched Semblance using a 15-Hz matching filter, and by the Wavelet Semblance (only the scales corresponding to 20, 15, 10 and 5-Hz are illustrated).

### Conclusions

This work presents a new coherency functional based on the Continuous Wavelet Transform for the velocity analysis of seismic reflection data affected by non-stationary signals. The Wavelet Semblance functional has been applied to a synthetic and to a field example. Both experiments demonstrate the capability of the Wavelet Semblance functional to explore efficiently the time-velocity-frequency
space producing a complete analysis that takes into account the occurrence of non-stationary signals. The results show that the Wavelet Semblance detects the weak attenuated reflections (i.e. sub-basalt reflections), that it produces high-resolution velocity spectra and that it is robust against noise. Finally, all the experiments are applied on raw data to stress the advantage, given by the Wavelet Semblance, to detect weak signals in an early stage of the processing.

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References


