Introduction

Due to different sedimentary environments, the realistic formation would show different electrical anisotropic characterizations. In general, only electrical anisotropy in layered unrotated media is considered, which is not always available in the practical application. Thinly laminated sand-shale sequences typically encountered in a low-energy environment belongs to this situation. It has the same resistivity in any horizontal direction but a different resistivity in the vertical direction, showing a classical 1D model (Wang et al., 2006; Zhong et al., 2008; Pardo and Torres Verdin, 2015; Wang et al., 2015, 2017). In order to differentiate it from other electric anisotropic media, it can be named as VTI media, i.e. transversely isotropy including a vertical symmetry principle axis. Formations that encountered in a relatively high energy environment, such as fluvial and desert environments, easily exhibit crossbedding anisotropy (Anderson et al., 1998, 2001; Wang et al., 2015, 2017). It shows transverse isotropy including a tilted symmetry principle axis, named as TTI media. TTI formations can be seen as the rotated uniaxial anisotropic media. Besides, reservoirs might have fractures because of the overburden pressure. Fractured formations show biaxial anisotropic characterization (Georgi et al., 2008). Similarly, it can be also divided into two types, unrotated and rotated biaxial anisotropic media, respectively indicating the formation has vertical fractures and tilted fractures.

Complicated anisotropic characteristics result in complicated electromagnetic logging responses. In order to accurately evaluate the formation, it is of great importance to investigate electromagnetic logging measurements under different anisotropic formation. It has been proved that Multi-component induction logging technology is quite sensitive to the electrical anisotropy. With its widespread availability, it is expected to solve more complicated electrical anisotropy by using this technology.

In this paper, based on the work of previous researchers, by using Dyadic Green’s function, we developed a semi-analytic forward algorithm to model the triaxial induction responses in layered arbitrary anisotropic media. Not only the tool orientation angles but also the media orientation angles are considered in the formula derivation. Anisotropic dip and anisotropic azimuth are used to represent the media orientation, indicating the anisotropic characterizations. Through numerical simulation, it can be found that triaxial induction responses are seriously affected by the anisotropic dip and anisotropic azimuthal other than formation resistivity, relative borehole dip and so on. After detailed discussion, some important phenomena have been found, for example, a critical borehole dip angle and a critical anisotropic dip is existed in the rotated anisotropic formation. Before and after these critical angles, the variation of coaxial magnetic components follow different courses. Besides, when anisotropic azimuth is zero, the critical borehole dip angle and critical anisotropic dip is easy to be determined.

Method and/or Theory

In the anisotropic media without rotation, the complex conductivity tensor $\bar{\sigma}$ in the Cartesian system of coordinates is expressed as:

$\bar{\sigma} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$

(1)

Figure 1

Geometry of anisotropic media with arbitrary orientation

In the uniaxial anisotropic media, $\sigma_x = \sigma_y \neq \sigma_z$. In the biaxial anisotropic media, $\sigma_x \neq \sigma_y \neq \sigma_z$. Figure 1a and Figure 1b respectively shows the unrotated media and rotated media. Anisotropic dip $\psi$
(corresponding to z-axis) and anisotropic azimuth $\chi$ (corresponding to x-axis), as shown in Figure 3, are used to represent their anisotropic characteristics. Essentially, $\psi$ and $\chi$ represent the formation orientation.

The conductivity tensor $\overline{\sigma}$ in an anisotropic media could be expressed as:

$$\overline{\sigma} = R \overline{R} \overline{\sigma} R R$$

where:

$$\overline{R}_\nu = \begin{bmatrix} \cos (\psi) & 0 & \sin (\psi) \\ 0 & 1 & 0 \\ -\sin (\psi) & 0 & \cos (\psi) \end{bmatrix} \quad \overline{R}_\nu = \begin{bmatrix} \cos (\chi) & \sin (\chi) & 0 \\ -\sin (\chi) & \cos (\chi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Dyadic green’s function is introduced to solve the following EM question with a time variation of $e^{-j\omega t}$:

$$\nabla \times \nabla \times \widehat{E} (r - r') - i \omega \mu_0 \sigma \widehat{E} (r - r') = - \nabla \times \widehat{M} (r - r')$$

where $\mu_0$ is the magnetic permeability of the air, $\omega$ is the angular frequency, $r$ and $r'$ are the position vectors of receiver and transmitter, $\widehat{M} (r - r')$ represents the magnetic-source, $\overline{\sigma}$ is the complex conductivity tensor.

After tedious derivation, it can be finally obtained the electrical field expression:

$$\widehat{E} (r) = \frac{i}{(2\pi)^2} \int \frac{d^3k}{(2\pi)^3} \left[ \alpha^*_{\nu} (\overline{e^*_\nu})^T - \frac{\lambda^*_{\nu}}{a \left( k_{\nu}^x - k_{\nu}^y \right) \left( k_{\nu}^x - k_{\nu}^y \right) \left( k_{\nu}^x - k_{\nu}^y \right)} \right] \cdot (\nabla \times \overline{\mu} (k)) \, dk \, dk'$$

where

$$\overline{\alpha} = \begin{bmatrix} \lambda_{\alpha}^* \left( \overline{e}_{\alpha}^* \right)^T \\ -a \left( k_{\alpha}^x - k_{\alpha}^y \right) \left( k_{\alpha}^x - k_{\alpha}^y \right) \left( k_{\alpha}^x - k_{\alpha}^y \right) \end{bmatrix} \quad z \geq z'$$

$$\overline{\alpha} = \begin{bmatrix} \lambda_{\alpha}^* \left( \overline{e}_{\alpha}^* \right)^T \\ -a \left( k_{\alpha}^x - k_{\alpha}^y \right) \left( k_{\alpha}^x - k_{\alpha}^y \right) \left( k_{\alpha}^x - k_{\alpha}^y \right) \end{bmatrix} \quad z < z'$$

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$$k_{\alpha}^x = \left( k_{\alpha}^x, k_{\alpha}^y, k_{\alpha}^z \right)$$

$$k_{\alpha}^y = \left( k_{\alpha}^x, k_{\alpha}^y, k_{\alpha}^z \right)$$

$$q = \text{upward or downward}$$

$\overline{\alpha}^*_{\nu}$ and $\overline{\alpha}^*_{\mu}$ could be seen as the amplitude vectors of type I and type II waves. $k_{\alpha}^x$ and $k_{\alpha}^y$ are wave vectors of type I and type II wave. $\overline{e}_{\alpha}^*$ and $\overline{e}_{\alpha}^*$ are unit vectors in the direction of the electric field. Since the magnetic field and the electrical field satisfy the following relationship:

$$\overline{H}_c = \frac{1}{i \omega \mu_0} \nabla \times \overline{E}$$

Therefore, it can be easily obtained the magnetic field by using equation (11).

As for the layered formation, the generalized reflection matrices (GRM), $\overline{R}_{L+1}$, and generalized transmission matrices (GTM), $\overline{T}_{L+1}$, between layer L and L+1 are introduced (Chew, 1995):

$$\overline{R}_{L+1} = \overline{R}_{L+1} + \overline{T}_{L+1} e^{\overline{U}_{L+1} d_{L+1}} R_{L+1} T_{L+1}$$

$$\overline{T}_{L+1} = \left( \overline{I} - e^{\overline{U}_{L+1} d_{L+1}} R_{L+1} e^{\overline{U}_{L+1} d_{L+1}} \overline{T}_{L+1} \right) e^{\overline{U}_{L+1} d_{L+1}} T_{L+1}$$

where $\overline{K}_{L+1} = \text{diag} \left[ k_{L+1}^x, k_{L+1}^y, k_{L+1}^z \right]$. $q = u, d$ and $d_{L+1}$ is the thickness of layer L+1.
By using equation (11), (12) and (13), magnetic field in layered arbitrary anisotropic media could be calculated.

Examples

The operation frequency and the spacing of triaxial induction tool used in this section are 26 kHz and 40in respectively. Six two-layer formation models are used. The borehole dip \( \alpha \) is fixed at 45\(^\circ\), the borehole azimuth \( \beta \) and orientation \( \gamma \) are both 0\(^\circ\). To investigate the influences of different electrical anisotropy separately, all the top layers are assumed as isotropic media. As for the bottom layer, varied types of anisotropic media are considered, including unrotated (\( \psi = 0\)\(^\circ\)) and rotated anisotropic media (\( \psi \neq 0\)\(^\circ\)). Table 1 shows the corresponding formation parameters.

Different electric anisotropy of the bottom layer makes the triaxial induction tool show different responses, as shown in Figure 2 and Figure 3. It can be found that in the rotated anisotropic media, no matter uniaxial or biaxial anisotropy, Hxx reaches the maximum and Hzz reaches the minimum. Before and after \( \psi, \gamma \). Hxx and Hzz follow different monotonous variation. Hxx increases at first and then decreases, whereas Hzz decreases at first and then increases. Furthermore, the responses near the boundary are seriously affected by different \( \psi \) and \( \gamma \).

### Table 1. Formation parameters of uniaxial and biaxial electrical anisotropic media

<table>
<thead>
<tr>
<th>No.</th>
<th>Top layer</th>
<th>Conductivity (S/m)</th>
<th>Bottom layer</th>
<th>Bottom layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Uniaxial anisotropy</td>
<td>Biaxial anisotropy</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>( \sigma_{1x} = \sigma_{1y} = \sigma_{1z} = 1.0 )</td>
<td>( \sigma_{2x} = 0.25; \sigma_{2y} = 0.125 )</td>
<td>( \sigma_x = 0.25; \sigma_y = 0.125; \sigma_z = 0.0625 )</td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>( \sigma_{1x} = \sigma_{1y} = \sigma_{1z} = 1.0 )</td>
<td>( \sigma_{2x} = 0.25; \sigma_{2y} = 0.125 )</td>
<td>( \sigma_x = 0.25; \sigma_y = 0.125; \sigma_z = 0.0625 )</td>
<td></td>
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<tr>
<td>c</td>
<td>( \sigma_{1x} = \sigma_{1y} = \sigma_{1z} = 1.0 )</td>
<td>( \sigma_{2x} = 0.25; \sigma_{2y} = 0.125 )</td>
<td>( \sigma_x = 0.25; \sigma_y = 0.125; \sigma_z = 0.0625 )</td>
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<tr>
<td>d</td>
<td>( \sigma_{1x} = \sigma_{1y} = \sigma_{1z} = 1.0 )</td>
<td>( \sigma_{2x} = 0.25; \sigma_{2y} = 0.125 )</td>
<td>( \sigma_x = 0.25; \sigma_y = 0.125; \sigma_z = 0.0625 )</td>
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<tr>
<td>e</td>
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<tr>
<td>f</td>
<td>( \sigma_{1x} = \sigma_{1y} = \sigma_{1z} = 1.0 )</td>
<td>( \sigma_{2x} = 0.25; \sigma_{2y} = 0.125 )</td>
<td>( \sigma_x = 0.25; \sigma_y = 0.125; \sigma_z = 0.0625 )</td>
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</tbody>
</table>

**Figure 2** Triaxial induction responses with different bottom rotated uniaxial media

**Figure 3** Triaxial induction responses with different bottom rotated biaxial media
Conclusions

The realistic formation normally present different anisotropic characteristics, rotated or unrotated, uniaxial or biaxial. Based on Dyadic Green’s function, we presented a fast forward algorithm for computing model triaxial induction responses in an arbitrary anisotropic formation. Anisotropic dip \( \psi \) and anisotropic azimuth \( \gamma \) are used to represent the rotated and unrotated anisotropic media. Numerical simulation is used first to testify the feasibility and robustness of our method.

In the rotated media, regardless of uniaxial or biaxial anisotropy, the variation of coaxial magnetic components with borehole dip and anisotropic dip is not monotonous. There exist a critical borehole dip \( \alpha_c \) and a critical anisotropic dip \( \psi_c \). Before and after these two critical angles, coaxial magnetic components follow different monotonicity.

The proposed algorithm and the discussion in this paper significantly supplement the understanding of triaxial induction responses in complicated anisotropic media. Clearly recognition of triaxial induction logging responses in arbitrary anisotropic media would assist us in reducing the resistivity logging interpretation error.

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References


