Introduction

While seismic data quality has continually improved to keep pace with the oil and gas industry’s need to image deep and smaller targets beneath complex overburden, there are remaining challenges to overcome. Even with the most advanced imaging algorithms, like reverse time migration (RTM) (Baysal et al., 1983; McMechan, 1983), subsalt images from Gulf of Mexico suffer illumination problems, and the pre-salt images from Brazil require even higher resolution. These challenges drive the development of new seismic acquisition designs, processing techniques and depth imaging and velocity modelling algorithms and workflows.

Mathematically, RTM is an adjoint operator instead of an inverse process of the model parameters. As a consequence, the RTM images often suffer from migration artefacts, limited resolution, and unbalanced amplitudes (Wang et al., 2016). Least-squares migration (LSM) was proposed (Lailly 1983; Lambaré et al., 1992; Jin et al., 1992; Nemeth et al., 1999) to compute the inverse of the modelling operator. The current LSM algorithms could be classified into two categories: image-domain methods which approximate the Hessian matrix by migration deconvolutions (Hu et al., 2001; Yu et al., 2006; Lecomte 2008), and data-domain methods which iteratively inverse the forward modelling operator. We focus on the data-domain least-squares RTM (LSRTM).

The conventional data-domain LSRTM algorithms work on stacked images and thus assume a correct velocity model. On synthetic data where we accurately know the velocity models, they demonstrate the capability to produce high-fidelity amplitudes, higher resolution, and fewer artefacts (e.g. acquisition footprints, migration ambiguity artefacts etc.). However, for real data applications, the velocity models are generally estimated from model building tools, and velocity errors are always present in the migration models.

We analyse the challenge for 3D real data applications and propose an efficient LSRTM scheme for practical applications. Our implementation is applied to common-shot gathers and achieves our goals of better illumination and higher resolution images. Our scheme has several advantages. First, it utilizes gathers instead of stacked images and therefore is robust and has the potential to tolerate mild velocity errors. Second, it preserves all the original focusing information and offers a better chance for an improved image. Finally, the high-fidelity common-image gathers (CIGs) from LSRTM may provide more reliable inputs for velocity model building and quantitative AVO analysis. We use a real data example from Gulf of Mexico to demonstrate the effectiveness of the scheme.

An efficient LSRTM scheme in common-shot gather domain

The objective function for conventional LSRTM which uses the stacked image is usually defined as:

\[ J_{stack} = \frac{1}{2} \sum_{x_r} \sum_{x_s} \sum_t ||d_{obs}(x_r, t, x_s) - d_{syn}(x_r, t, x_s; m_{stack})||^2 \quad (1) \]

where \( m_{stack} \) is the stacked image, \( d_{obs}(x_r, t, x_s) \) represents the acquired data, and \( d_{syn}(x_r, t, x_s; m_{stack}) \) represents the synthetic data. \( x_s \) and \( x_r \) are the shot and receiver locations. \( t \) denotes time.

According to equation (1), the conventional LSRTM uses the stacked image for de-migration to generate the synthetic data, which makes it difficult to converge if the velocity is inaccurate. Figure 1 demonstrates the sensitivity of LSRTM to velocity errors. With a correct velocity, the seismic events in CIGs (Figure 1a) are flat and the stacked signal (Figure 1b) is good as well (correct wavelet and true depth location). With velocity errors, the seismic events in CIGs (Figure 1c) are curved and will result in a poor stacked signal (Figure 1d) with distorted phase and amplitude. If this distorted stacked signal is used for LSRTM, the generated synthetic data will also have the wrong phase and amplitude, and LSRTM will fail to converge. In this case, using stacked images would be detrimental to the convergence.
Figure 1 Example of velocity error effects on CIGs and stacks. A correct velocity generates flat CIGs (a), and the stacked signal (b) with correct phase and amplitude. A slightly wrong velocity generates curved CIGs (c), and the stacked signal (d) with distorted phase and amplitude.

One option to reduce the sensitivity of LSRTM on velocity models is to implement LSRTM in the gather domain (Hou and Symes, 2015; Wang and Xu, 2017). In theory, the LSRTM will converge in any migration gather domains but different gather types yield different numerical efficiency and requirement of resources. We investigated several types of gathers, such as surface offset gathers, subsurface offset-shift or time-shift gathers, subsurface angle gathers, and common-shot gathers. For 3D real data applications, LSRTM with common-shot gathers is the most efficient one and needs the least memory/disk.

To implement LSRTM with common-shot gathers, we use the following objective function for LSRTM with each shot:

$$J_{shot} = \frac{1}{2} \sum_{x_r} \sum_{t} \| d_{obs}(x_r, t) - d_{syn}(x_r, t; m_{shot}) \|^2$$  \(2\)

where \(m_{shot}\) is the image obtained only using data from a shot located at \(x_s\). While equation (1) uses the stacked image for modelling/de-migration to obtain the synthetic data in each iteration, equation (2) only uses the single shot image for modelling/de-migration.

The following is the workflow for LSRTM with common-shot gathers, for each shot:

(a) Migrate the acquired data \(d_{obs}(x_r, t)\) to obtain an image \(m_{shot}\);

(b) Use the obtained image \(m_{shot}\) as input and do synthetic modelling (de-migration) to get the synthetic data \(d_{syn}(x_r, t; m_{shot})\);

(c) Compare \(d_{syn}(x_r, t; m_{shot})\) with \(d_{obs}(x_r, t)\). Iteratively migrate the data residual to update the image \(m_{shot}\) until the residual is acceptably small.

The output from LSRTM will be CIGs indexed by shot numbers. These gathers can be used for velocity analysis or converted to angle gathers for quantitative analysis (Duveneck et al., 2019). Optimal stacked images can be obtained by correcting the event curvatures in the output CIGs from LSRTM.

There are several advantages of working with common-shot gathers. First, a common-shot gather is a subset of the seismic data and working on the subset has a better chance to converge. LSRTM with gathers can tolerate mild velocity errors. Second, it reduces the computational cost as dealing with a single shot means a limited aperture. Third, the memory/disk demand is small, because we do not need stacking or further processing to generate gathers for LSRTM to use in the de-migration step. Furthermore, no cross-node communication between different shots is needed and it is easy for parallelization.

Real data examples

We applied our LSRTM scheme to a real dataset from the Gulf of Mexico. The velocity model is TTI anisotropic, and the models are estimated by tomography and full waveform inversion (Tarantola 1984;
Pratt, 1999). We select 6 shot lines, including 663 shots, for migration at the target area. The migration maximum frequency is 18 Hz. The maximum depth is 18 km.

We first apply LSRTM for each shot and generate the CIGs indexed by shot numbers. As the velocity model is not perfect, the seismic events in the CIG gathers are not very flat, and thus a straightforward stack would reduce the resolution. In addition, we developed a stacking method to automatically handle the subtle curvatures and generate the stacked images. Figure 2 displays the LSRTM image and its corresponding RTM comparison from the inline direction. Figure 3 displays the images from the crossline direction. Compared to the RTM images, LSRTM shows higher resolution, better illumination, and improved continuity (for example, the events shown by the red arrows in Figures 2 and 3). We observe both improvements for the sediment area above the salt body and for the challenging areas beneath the salt.

**Figure 2** Gulf of Mexico data example. Inline comparison. (a) RTM image, (b) LSRTM image.

**Figure 3** Gulf of Mexico data example. Crossline comparison. (a) RTM image, (b) LSRTM image.

**Conclusions**

We propose an efficient LSRTM scheme which works on common-shot gathers. It handles velocity errors for real data applications. We run LSRTM for each single shot and generate CIGs. Comparing with other types of CIGs, it is the most efficient and has the minimal memory/disk requirement. Not requiring interaction between shots or communication between nodes makes it easy for parallelization. Thus, it is practical for 3D real data applications. With an optimal stacking algorithm, we can obtain high-quality stacked images from the CIGs. The output CIGs can be used for velocity model building and AVO analysis as well. Although we demonstrate the method with common-shot gathers, the same scheme works for common-receiver gathers based on the reciprocity principle.
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References