Delineating a cased borehole in unconsolidated formations using dipole acoustic data from a nearby borehole

Introduction

Single-well acoustic imaging has been used for decades in borehole geophysical exploration to characterize geological structures outside the borehole (Hornby, 1989; Tang and Patterson, 2009). This technology can effectively detect reflectors tens of meters away from borehole. Besides the geological structures, an existing borehole around the measurement borehole is also a target of special interest for the imaging application. For example, accurate delineation of the trajectory of the existing borehole is important for safe drilling in densely drilled areas, especially in very soft formations frequently encountered in drilling a poorly consolidated sand in shallow water marine environment. Additionally, most of the boreholes in the shallow soft formations are cased, and the effects of casing and cement should be considered (Peng, 1994). The main objective of this study is to develop a theoretical analysis for detecting a cased borehole using dipole acoustic (P-wave) data from a nearby well in the very slow formation environment.

In what follows, we first develop the theoretical formulation for analyzing the interaction between the cased borehole and the incident elastic wave from another borehole. Then, in conjunction with the method of steepest descent and the reciprocity theorem, an asymptotic solution for the P-wave data in the measurement well is derived. For validation, the analytical solution result is compared with that from a 3D finite difference modeling. Afterwards, the data characteristics are analyzed to show the advantages of P waves over S waves for the borehole detection in the slow formation. Finally, we present a field data example to support the validity of our result and demonstrate the significance of the application.

THEORETICAL ANALYSIS

The schematic of the borehole detection model is illustrated in Figure 1. A dipole acoustic tool is placed in a newly drilled (uncased) borehole called the measurement hole, which is parallel to the nearby cased borehole, called the target hole. The two boreholes are separated by a distance \( r_\text{0} \). The dipole transmitter of the tool radiates an elastic wave into the formation (modeled as a very slow formation). After incidence on the target hole, the wave is scattered backward to the measurement hole and is received by an array of receivers in the borehole fluid. The incident elastic wave, as radiated from the measurement hole, is a spherical wave. The wave spectrum can be generally written as (Tang et al., 2016)

\[
Iwv(\omega) = S(\omega) \cdot \frac{RD(\omega; \theta, \phi)}{4\pi \nu^2} \left( \frac{e^{j\omega R/\nu}}{R} \right)
\]

where the expression in the brackets is typical of the spherical wave with a geometric spreading factor \( 1/R \); \( \rho \) is formation density; and \( \nu \) is wave velocity of the formation, which, depending on the type of incident wave. \( S \) and \( RD \) are the spectrum and radiation function of the acoustic source in the measurement hole. In fact, \( RD \), a dimensionless quantity, is also called the far-field radiation directivity. For the cylindrical geometry of the target borehole, it is convenient to express the spherical incident wave on equation 1 as a series summation of cylindrical waves using Sommerfeld integral and Graf’s addition theorem (Tang et al., 2016)

\[
\frac{e^{j\omega R/\nu}}{R} = \frac{1}{\pi} \int_{-\infty}^{\infty} \sum_{n=0}^{\infty} e_{n}K_{n}(k_{0}r_{0})I_{n}(k_{r}r)\cos(n(\varphi-\varphi_{0}))e^{jk(z-z_{0})} dk
\]

where \( e_{n} \) is 1 for \( n = 0 \) and 2 for \( n > 0 \); \( I_{n} \) and \( K_{n} \) are the \( n \) order modified Bessel functions of the first- and second-kind, respectively; \( (r_{0}, \varphi_{0}, z_{0}) \) are the cylindrical coordinates of the borehole source; \( k \) is the axial wavenumber, \( k_{r0} = \sqrt{k^2 - \omega^2/\nu^2} \) is the radial wavenumber. The incident wave on the
cased borehole induces wave motion in the fluid inside casing and in the layered casing-cement structure surrounding the borehole. The wave amplitude coefficients of elastic-waves in the borehole fluid and layer m are determined by the boundary condition at the fluid-casing interface \( r = r_1 \)

\[
H \times \left[ A_n^f B_n^{(m)} C_n^{(m)} D_n^{(m)} \right] r = d \text{ and } H \times \left[ A_n^f B_n^{(m)} C_n^{(m)} D_n^{(m)} \right] r' = d'
\] (3)

where \( H \) is a 4x4 matrix calculated using the propagator matrix technique, \( d \) and \( d' \) are 4 × 1 vectors whose elements are calculated from the incident wave and evaluated at the inner casing interface (Peng, 1994). Solving equation 3 to determine the coefficients of the scattered wave \( m = 3 \) from the target borehole for the P-wave incidence. The scattered wavefield will be measured by the receiver array in the measurement hole. For this case the radial distance \( r \sim r_0 \) from the target borehole is large compared to the wavelength such that \( |k_p r| \gg 1 \) and \( |k_p r_0| \gg 1 \), the steepest descent method can be applied to replace the wavenumber integration with the steepest solution:

\[
u_i'(\omega, r, \varphi, z) = \sqrt{\frac{\pi \omega \sin^3 \delta}{2 \alpha^3 (r + r_0)}} \sum_{n=0}^{\infty} B_n^{(3)}(\omega, k_0) \cos(n \varphi)
\]

\[
+ B_n^{(3)}(\omega, k_0) \sin(n \varphi)) e^{i \omega (r' + r \sin \delta) (z - z_0)/4}
\]

where \( k_0 = \omega / c_0 \cos \delta \) and \( R' = r^2 + (z - z_0)^2 \). The reception of the scattered wave in the measurement hole can be calculated by using the source-receiver reciprocity (Xu et al., 2019). Receiving the incident wave inside borehole fluid is governed by a borehole reception function

\[
RWV(\omega) = RC(\omega) \cdot u_i'(\omega, r, \varphi, z)
\]

where \( RWV \) denotes the received wave signal inside borehole. And the reception function \( RC(\omega) \) is equivalent to the borehole radiation function \( RD(\omega) \).

**RESULT VALIDATION AND APPLICATION**

Figure 2 compares the calculated fluid displacement signals at the borehole center from the analytical (solid curve) and numerical (dots) modeling. For the model of two parallel boreholes and the coordinates system of Figure 1, the source is at \( z_0 = 0 \) and the source and receivers are co-located at the measurement borehole axis \( r = r_0 = 5 \text{m} \). The elastic parameters of the two boreholes and the formation are given in Table 1. The agreement between the analytical and numerical results is excellent. The analytical solution takes only a few seconds to get the result of Figure 2, while the 3D finite
difference modeling needs many hours of calculation. Therefore, our analytical solution provides a fast algorithm for forward modeling the elastic-wave interaction between two boreholes.

We now further discuss the advantage of P waves over S waves for the target borehole detection in very slow formations. Figure 3 compares the calculated P-, SH-, and SV-wave radiation/reception directivity patterns for the measurement borehole and the formation parameters of Table 1. The result shows that the amplitude of the $RD_p$ directivity (black in Figure 3a) is much larger than those of the $RD_{SH}$ (red in Figure 3b) and $RD_{SV}$ (black in Figure 3b) directivities. Of particular interest is at the angle range around $\delta=90^\circ/270^\circ$, where wave radiation and incidence occur. $RD_{SV}$ is null while $RD_p$ and $RD_{SH}$ attain their respective maximum value, with the $RD_p$ maximum about 20 times larger than the $RD_{SH}$ maximum, revealing that the P-wave predominates the dipole source radiation and reception in the very slow formation.

To verify the above analysis, the modeling is repeated for the SH-wave incidence of the same model to calculate the received fluid displacement normal to the wave incidence plane. The result is compared to that of the P-wave scenario in the time range from 5 to 28ms, as shown in Figure 3(c). In this figure, the 10 to 20ms interval is skipped because the very slow SH wave arrives around 22ms. The SH-wave amplitude is multiplied by a factor of 30 in order to visualize this very weak wave event. One can therefore conclude that it is more effective and advantageous to use P waves than S waves for detecting a target borehole with a very slow formation.

![Figure 3](image.png)

**Figure 3** P-wave (a) and SH- and SV-wave (b) radiation/reception directivity patterns in a very slow formation for a 3 kHz dipole source. (c) Comparison of the P- (early arrival) and SH- (later arrival) waves scattered from the target borehole. The amplitude of SH-wave is increased by a factor of 30 to visualize the very weak signal.

As an important application of the theoretical analysis result, Figure 4 demonstrates an example of cased borehole delineation in the unconsolidated formation of shallow marine environment. Panel 2 shows the variable density (VD) display of the dipole acoustic logging data of this interval. For this very slow formation, the early portion of the data shows mainly the high-amplitude P waves traveling along the borehole, from which the DTP curve of Panel 1 was derived. Using a wave separation technique to suppress the large amplitude borehole arrival, the wave signal from a nearby (cased) borehole can be visualized, as shown by the VD display in Panel 3. The wave event, after migration using the DTP data of Panel 1, gives the image of a steeply dipping reflector in the vicinity of the borehole (see Panel 5), the distance of the reflector ranging from 7.7m to 11.4m in this 50m depth interval. The well location...
maps of the drilling area confirmed this is a cased well near the logging well. For further verification, we performed a forward modeling to simulate the dipole acoustic data of Panel 2. The modeled scattered P wave in Panel 4 and its measured counterpart in Panel 3 have about the same arrival time confirming that the wave event in Panel 3 is the scattered wave from the nearby well. This field data example indicates that the P-wave data from dipole acoustic logging can indeed be utilized to delineate the nearby (cased) well in unconsolidated soft formations.

![Field example of using dipole acoustic data (Panel 2) to delineate a nearby borehole target (image in Panel 5) for an unconsolidated formation. The P-wave signal scattered from the target hole is shown in Panel 3 and confirmed by the modeling result in Panel 4.](image)

**Figure 4** Field example of using dipole acoustic data (Panel 2) to delineate a nearby borehole target (image in Panel 5) for an unconsolidated formation. The P-wave signal scattered from the target hole is shown in Panel 3 and confirmed by the modeling result in Panel 4.

**Conclusions**

By analyzing the interaction of the elastic wave propagation with a cased borehole in the very slow formation environment, the effects of wave radiation, reception, and borehole scattering on the dipole acoustic measurement in the nearby well can be correctly modeled. The combination of the steepest descent solution and the reciprocity theorem provides a fast method for theoretical modeling. The accuracy of the modeling result was verified by comparison with the 3D finite difference modeling result. The result shows that the dipole-generated P wave, because of its dominance over the S wave, is advantageous to use for imaging a near-well borehole target in the very slow formation situation. The result of this study can be used to provide a method for delineating a nearby borehole target using dipole acoustic logging in the shallow unconsolidated sediments.

**References**


