Introduction

Gravity field data is a very meaningful geophysical data for mineral and petroleum exploration, geological structure study, development of urban sub-surface space. The inversion of gravity data is one of the most important topics in quantitative interpretation of practical data, since construction of density contrast models could increase the amount of information that can be achieve from the gravity data (Rezaie et al., 2017). The major difficulties are the inherent non-uniqueness and efficiency. According to Gauss’s theorem, there are infinite equivalent source distributions that produce the same measured gravity data. Non-uniqueness of inverse problem solution can be overcome by applying a numerical constraint. Several methods have been developed to solve this non-uniqueness problem. For example, Farquharson (2008) used L1 measure of horizontal, vertical and diagonal derivatives in the model objective function to recover dipping structures and models which have angled interfaces in the version of 2-D MT data and 3-D gravity data. Another difficulty in gravity inversion, efficiency, is related to large processing time due to large data sets over large exploration regions and the necessary increase in model parameters. There are two main research directions to increase the inversion efficiency, effective algorithms and computer hardware. For example, the frequency domain calculation is used to increase the efficiency, such as Fourier transform, wavelet transform and so on. The mathematical optimization calculation method is applied to increase the forward model calculation and inverse solution. And with development of computer hardware, GPU is used to speed up the potential field data inversion. In this paper, inversion accuracy and calculation efficiency are our main study directions. We choose to use an approximate L0 norm to obtain sparsity inversion results. And we use a DCT algorithm to project the large scale gravity problem into the small subspace. In this new small space, the calculation efficiency of gravity inversion is improved. First, sparsity gravity inversion using an approximated L0 norm is described, and then the reweighted regularized method using DCT algorithm is discussed. Also a Tikhonov regularization algorithm is studied by GCV algorithm. Finally, the capabilities of the reweighted regularized Lanczos bidiagonalization method are illustrated by its application to the inversion of a synthetic data set and to the 3D inversion of gravity data from the Mobrun ore body, northeast of Noranda, Quebec, Canada.

Basic theory

To solve the gravity inversion problem, we divide the sub-surface space in the research region into small blocks with constant density contrast. The density contrasts within each cell is a known parameter and gravity response of each cell is calculated using the formula discussed by Boulanger and Chouteau (2001). The forward modeling is discrete by a linear relationship.

\[ Ax = b \] (1)

Here, \( b \) is the observed gravity data, and \( x \) is the sub-surface density value of each cell. A is forward operator matrix that maps the physical parameters space into the data space. And the observed gravity is always contaminated by the noise. The solution of eq.(1) is not accurate. In general gravity inversion, the sub-surface space is bigger than the surface data space. Therefore, the unknown density cell number is much bigger than the observed data. The gravity inverse problem is ill-posed and the solution is unstable. We solve this problem by the minimization of Tikhonov parametric functional:

\[ \text{min} \quad \Phi = \Phi_b + \lambda \Phi_x \] (2)

where the misfit functional is defined as

\[ \Phi_b = \| W_b (b - Ax) \|^2 \] (3)

and the stabilizer function is always defined as

\[ \Phi_x = \| W_x (x - x_{ref}) \|^2 \] (4)

\( \lambda \) is a regularization parameter which controls relative balance between \( \Phi_b \) and \( \Phi_x \). In this paper, we will use GCV algorithm to get a suitable regulation parameter to obtain a stable solution. In eq. (3), \( W_b \) is the data weighting function usually consisting of the estimated data-error variances. It is given by \( W_b = \text{diag}(1/\sigma_1^2, ..., 1/\sigma_N^2) \), where \( \sigma_i \) stands for the standard deviation of noise in the \( i \)th observed datum. A stabilizer function describes the model characteristics. And a stable and unique solution is obtained. \( W_x \) is the depth weighting matrix to compensate lack of the gravity data sensitivity to the
deeper model parameters. It is defined by \( W_x = \text{diag}(1/(z_1^{\beta}), \ldots, 1/(z_M^{\beta})) \). Here, \( z_j \) is the depth of \( j \)-th model parameter and \( \beta \) is used for gravity data.

The standard Tikhonov regularization technique produces a smoothing effect on the resulting solution as it determines smooth features while blurring jumps in the regularized solution, a characteristic known as over-smoothing (Gholami and Siahkoohi, 2010). To obtain a block inversion result, we use a sparsity constraint, L0 norm, into equ. (2). We get a new target equation.

\[
F_{\sigma}(x) = \|x\|_0
\]

(5)

With this new constraint equation, we can obtain a block and stable solution. Here, L0 norm of \( x \) is defined by

\[
\|x\|_0 = \lim_{\sigma \to 0} \|x\|_0^\sigma = \lim_{\sigma \to 0} \left( \sum_{i=1}^M |x^i|^2 \right)^{1/2}
\]

(6)

We can easily understand the specific meaning of L0 norm. However, a minimum of equation (6) or (7) is very hard. It is well known that the minimization of zero norms is NP-complex problem in combinatorial optimization. It is difficult to solve this problem in polynomial times. Hence this model is doomed to be infeasible in practice.

An effective algorithm need be selected to solve this problem. From many studies (Mohimani et al., 2008), a smooth approximated L0 norm is useful for this problem. An approximate smooth function is studied to replace the L0 norm.

\[
f_\sigma(x) = |x|/|x| + \sigma
\]

(7)

The equation (5) becomes

\[
F_{\sigma}(x) = \sum_{i=1}^M f_\sigma(x^i) \to \min
\]

(8)

One can see that when \( \sigma \) is larger, the approximation function shows smoother but not closer to L0 norm, and vice versa. Hence, we start with large enough \( \sigma \), to include all potential solutions. Then with iterations, the \( \sigma \) is decreased and the approximation function becomes closer to L0 norm, to obtain a more accurate solution. How to resolve equation (8) is an important part in the potential field data inversion. We use two parts to obtain the accurate inversion results. Quasi-Newton algorithm is applied to obtain a sparsity solution, and regularization SVD algorithm is used to project results back into feasible set. At each iterative step any cell density value Finally, to obtain a geological and geophysical meaning result, a density constraint is applied to that falls outside practical lower and upper bounds, \([x_{\min}, x_{\max}]\), is projected back to the nearest constraint value, to assure that reliable subsurface models are recovered.

Now, we have solved the non-uniqueness and unstable problem in the gravity inversion. Next, we pay attention on the calculation efficiency of gravity inversion. Many algorithms are applied to resolve this problem, such as optimal algorithm and matrix compression. They both improve the inversion efficiency. However, the compression resolves the essence of large scale data inversion. In this subsection, a compression algorithm is used to compress forward matrix. We all easily understand that the decreasing of forward matrix \( A \) may affect the accuracy of inversion. A compression algorithm should include both the desired basis vectors and some basis vectors that are not so important for the regularized solution, due to the noise. At the same time the significant basis vectors have not emerged in this sub-space before some undesired basis vectors appear. DCT algorithm is very suitable for this application. Using this algorithm, a large scale observed data and forward matrix can be reduced evidently. In this part, the DCT matrix is defined by

\[
W_c(k) = \sqrt{\frac{2}{N}} \cos \frac{(2n+1)k\pi}{2N},
\]

\[
k,n = 0,1,2,\ldots,N-1,
\]

(9)
here, \( N \) is the number of observed data or others, and \( c(k) \) is defined by

\[
c(k) = \begin{cases} 
1 & k = 0 \\
\frac{1}{\sqrt{2}} & k \neq 0 
\end{cases}
\]

Using DCT transformation, the main information of data will concentrate mainly in the low frequency region. We can improve the inversion efficiency.

**Real data: Robrum anomaly, Noranda, Quebec**

To illustrate the relevance of the approach for a practical case, a residual gravity data is acquired at the Mobrun sulfide body in Noranda, Quebec, Canada. The gravity anomaly is caused by a body of base metal massive sulfide, which has hosted volcanic rocks of middle Precambrian age. The previous geological information gives the special information. And the sub-surface density distributions of sulfide body and mineralized zone have been determined by drilling some borehole. Its location and density information are shown in Fig. 1. The top depth of anomalous body is about 17 m and its extension is 187m (Grant and West, 1965). And the corresponding rock density is 2.7 g/cm\(^3\), and the massive cherty pyrite is 4.6 g/cm\(^3\). In this region, the density of some rock far from the ore body will become small. Therefore, the max density contrast is 1.9 and the min is -0.1, which is accorded with the actual geological conditions. The real gravity dataset is composed by a regular grid of 37 × 31 data and the line and point space are respectively 20 m. Fig 2 A) give the gravity map constructed using dataset.

**Figure 1** The iterative curve for these inversion methods in this paper. A) shows the convergence curve of data fitting, B) shows convergence curve of the difference between inversion density model and true density model, C) gives the value of regularization parameter in each iteration.

**Figure 2** Gravity data and sub-surface density distribution. A) Gravity anomaly data of Mobrun deposit, AA' and BB' show the cross-section position, B) depth slice at \( z = -100 \) m through recovered density model, C) cross-section slice at \( y =320 \) m, D) cross-section slice at \( x = 360 \) m.

For the research of gravity data inversion, the whole sub-surface in this region has been divided into \( 37 \times 32 \times 15 \) cells of 20m in the x, y and z direction respectively. The real gravity has been inverted by our inversion method proposed in this paper, sparsity inversion using DCT compression algorithm. Depth slice map (at \( z = -100 \) m) of density model has been shown in Fig. 2 B). It shows the horizontal space trend of sulfide body, which is accord with the true distribution. In Fig. 2 C) and D), two cross-sections (at \( Y = 320 \) m and \( X = 360 \) m) of the recovered density distributions are discovered. We easily find the vertical distribution of ore body. To show the inversion results clearly, 3D views are shown in Fig 3. Fig 3. A) shows the position of ore body in this sub-surface space. And Fig 3. B) gives the characteristics of the spatial distribution of ore body. Lots of information is shown in this 3D view figure. From these inversion results, the depth of the top of the body is about 20 m and it extends to the depth of more than 180 m. Thus, the density distribution obtained by our new inversion method proposed in this paper is accord with the true information.
Figure 3 Different models of 3D view at Mobrun sulfide body. A) 3D density distribution of the whole sub-surface space, B) only the sulfide body for a cut off equal to 0.8 g/cm$^3$.

Figure 4 Data comparison A) show the predicted gravity anomaly map obtained from the inversion results, B) show the difference between observed gravity data and calculated gravity data, C) the profile gravity data at $y = 320$ m, D) at $x = 360$ m.

To show the availability of real data inversion results (Fig. 4), we explain the characteristics of inversion results from different perspectives of calculated data. The data misfits are very necessary to demonstrate the accuracy of inversion results. Overall, the theoretical calculated data and observed data fit each other well. The sparsity inversion using compression algorithm is efficient for practical application.

Conclusions

The sparsity inversion with discrete cosine transform compression using SVD and quasi-newton iteration algorithms has been presented. Using sparsity constraint in the inversion, the weight of anomalous parts in the sub-surface will increase and the accurate solutions are obtained. The inversion results are block, whose density boundary is clear. It is very suitable for the resource exploration. It has been found when the kernel multiplied by matrix of the orthonormal basis vector of DCT algorithm, the number of large forward matrix is decreased. Therefore, the required memory is also decreased and inversion process becomes faster. This DCT algorithm also can improve the condition number of forward matrix and then improve the numerical stability, which is very necessary for inversion problem. In the synthetic application, this new inversion requires 20% less memory. This compression algorithm applied in the inversion improves the speed and accuracy of gravity data inversion. Using the GCV method to estimate the regularization parameter for the full space and sub-space solution gives optimal regularization parameter and thus provide solutions that are satisfactory. This new presented inversion is practical and efficient and has been illustrated for the inversion of real gravity data.

References


