Introduction

By reason of viscoelastic properties of the formation medium, the seismic wave propagation through the near-surface suffers from strong absorption. The quality factor Q is a commonly used parameter for quantitatively characterizing the absorption and attenuation characteristics. Traditional Q estimation methods are based on the assumption that Q values are constant (frequency-independent Q) in the effective band interval, such as the widely used spectral ratio method (LSR) (Bath, 1974) and centroid frequency shift method (CFS) (Quan and Harris, 1997). However, theoretical studies of attenuation mechanisms (Müller. et al., 2010), laboratory measurements of reference samples and actual data processing (Zhan. et al., 2018) tend to suggest that Q values is dependent on frequency. Jeng (1999) proposed a power function relationship between Q and frequency for the effective frequency range of actual seismic data, proving that the linear regression used by the LSR would introduce systematic bias into the Q estimation. Li et al. (2016) analyzed power function’s error mechanism in detail, and obtained it was mainly from the exponential term of the power function and proposed a two-parameter regression estimation frequency-dependent Q method (TPM). However, the frequency-dependent Q is currently only in the actual seismic data calculation and laboratory rock sample testing process, and there are few reports on the laboratory seismic physical simulation.

Using ultrasonic to measure attenuation Q values under laboratory conditions originated the 20th century. Toksöz et al. (1979) analyzed the attenuation characteristics of dry samples and saturated samples in detail by using pulsed transmission waves, and proposed the method of measuring the Q value of samples by reference sample method for the first time. Until now, the method is still widely used in laboratories. However, Gao et al. (2018) compared the laboratory seismic pulse transmission method, the pulse transmission insertion method and the reflected wave method, and found that the Q value obtained by the pulse transmission method was strongly influenced by the diffraction, and the error exceeded 50% or more. For this, he proposed an improved method and reduced the error.

In this paper, the Q value of the reference sample aluminum is firstly tested to explain the frequency dependence of the Q value and the measured error of the LSR and CFS under laboratory conditions. Then, we use the pulse transmission method to measure the seismic wave attenuation of the laboratory and the reference sample method to measure the Q value of the epoxy resin as the measured sample. The two conventional methods (LSR and CFS) based on frequency-independent Q hypothesis and the TPM method estimating frequency-dependent Q value are compared, which further illustrates the dependence of seismic physical simulation attenuation measurements on frequency. Finally, the Q value obtained by the TPM is more accurate and stable on base of compensated result.

Estimate frequency-independent Q via traditional method

The spectral ratio method is to estimate the Q value based on a linear relationship between the logarithm of amplitude and frequency.

\[ l(f) = c - \frac{\pi f (t_2 - t_1)}{Q} \]  \hspace{1cm} (1)

Where \( l(f) = \ln \left( \frac{A(f, t_2)}{A(f, t_1)} \right) \), \( c \) can be regarded as a constant.

The centroid frequency shift method (CFS) is to estimate the Q value according to the decrease of the centroid frequency as the increase of the propagation time.

\[ Q = -\frac{\pi (t_2 - t_1) \sigma_1^2}{(f_1 - f_2)} \]  \hspace{1cm} (2)

Where \( f_1 \) and \( f_2 \) stand for the centroid frequency at time \( t_1 \) and \( t_2 \), respectively. \( \sigma_1^2 \) is the variance at time \( t_1 \) and \( t_2 \).

Estimate frequency-dependent Q via two-parameter regression
The frequency dependent Q is usually simplified to (Jeng, et al. 1999)
\[
Q^{-1}(f) = \alpha(f) \gamma
\]
(3)
where \( \alpha = Q_0^{-1} (1/f_0) \). \( \gamma \) is a constant in the interval (-1, 1), \( f_0 \) is the reference frequency, and \( Q_0 \) is the quality factor Q at the reference frequency. Based on this equation, the seismic attenuation defined by equation (1) becomes
\[
l(f) = c - \pi \alpha(f)^{\gamma + 1} \Delta t
\]
(4)
In this way, the seismic attenuation is no longer linear with frequency, and its slope also becomes frequency dependent, so the Q estimate translates to the following objective function solution.
\[
Obj = \min \sum f \left[ l(f) - c + \pi \alpha(f)^{\gamma + 1} \Delta t \right]^2
\]
(5)

Seismic physical model study

We use the pulse transmission method to measure the Q values of the aluminum and the epoxy resin samples on laboratory tests. The dimensions of the aluminum and epoxy resin are shown in Table 1. Main frequency of the transducer is 0.5MHz, and the sampling interval is 4.0\times10^{-8}s. Figure 1(a) and (b) show the measured waveform and the corresponding spectrum, respectively.

Table 1 The dimensions of aluminum and epoxy resin

<table>
<thead>
<tr>
<th>Sample</th>
<th>Epoxy resin No.1</th>
<th>Epoxy resin No.2</th>
<th>Aluminum No.1</th>
<th>Aluminum No.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension/mm</td>
<td>56.270</td>
<td>56.130</td>
<td>50.00</td>
<td>60.00</td>
</tr>
</tbody>
</table>

![Figure 1](a) Measured waveform. (b) The spectra of aluminium and epoxy resin samples.

The spectrums of aluminum in Figure 1(b) shows that there is a certain attenuation more or less for aluminum samples in reality, although aluminum is generally similar to non-attenuating sample under laboratory conditions (Pyra, et al, 1990). To verify the frequency dependence of the attenuation of the aluminum sample under laboratory conditions, the Q value of the aluminum No.2 is estimated by LSR and CFS methods using the aluminum No.1 as the reference sample. The estimated Q of LSR and CFS are close. However, the Q value of LSR and CFS for aluminum is too small obviously, because the Q value of aluminum sample is generally in the range of several thousands to hundreds of thousands (Zemanek and Rudnick, 1961; Gao 2018). We estimate the frequency-dependent Q value of the aluminum using TPM, shown in Figure 2(a), and expression is \( Q^{-1}(f) = 2.9068(f)^{0.9477} \). The Q value is 107790.68 corresponding to the frequency of 0.075MHz and 10240.83 corresponding to frequency of 0.975MHz, the Q value at the main frequency of 0.5MHz is 58401.18. The Q value of aluminum by TPM is reasonable. The estimated Q value of the aluminum in the laboratory proves that the frequency-dependent Q value estimated by TPM is more in line with the laboratory requirements, more accurate and more mature, compared with the Q value obtained by the LSR and CFS.

The above estimation of the Q value of the aluminum illustrates the applicability of the TPM to estimate the frequency-dependent Q value. To better verify the actual attenuation mechanism, we
estimate the Q value of epoxy resin samples No.1 and No.2. The LSR and CFS are used to estimate the frequency-independent Q and the results are shown in Table 2. The frequency-dependent Q values estimated by TPM are shown in Figure 2(b). The expression corresponding to epoxy resin No.1 and No.2 are \( Q^1(f) = 0.7478 \times 10^{-3}(f)^{2.495} \) and \( Q^2(f) = 2.2271 \times 10^{-3}(f)^{0.1661} \), respectively. The Q values vary between 40 and 65 and the Q value at the main frequency are 50.7879 and 50.6078.

**Table 2 Frequency-independent Q value obtained by LSR and CFS**

<table>
<thead>
<tr>
<th></th>
<th>Aluminum No.2</th>
<th>Epoxy resin No.1</th>
<th>Epoxy resin No.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSR</td>
<td>274.7826</td>
<td>33.6120</td>
<td>34.7935</td>
</tr>
<tr>
<td>CFS</td>
<td>240.1250</td>
<td>30.1579</td>
<td>33.2557</td>
</tr>
</tbody>
</table>

**Figure 2** (a) Frequency-dependent Q value for the aluminum No.2. (b) Frequency-dependent Q value for the epoxy resin No.1 and No.2.

To further verify the feasibility and effectiveness of estimating the frequency-dependent Q by TPM, we use the Q values obtained by the above three methods to compensate the attenuation using inverse Q filtering method of the stability factor (Wang 2002).

**Figure 3** Waveforms and spectrums of the epoxy resin measurement samples No.1 and No.2 compensated using Q values obtained by the TPM (top), the LSR (middle) and the CFS (Bottom).

Figure 3 shows the waveforms and spectrums of measurement samples epoxy resin No.1 and No.2 after compensating, whose Q values are estimated by TPM, LSR and CFS. We can see that the results of compensation using the Q values estimated by LSR and CFS are overcompensation, that is, the Q
value is too small. In comparison, the compensation using the Q value estimated by TPM is more consistent with the spectrum of the reference sample aluminum No.1, and there is no amplification noise from the compensated waveform, which proves that the TPM is accurate and effective.

Conclusions

Ultrasonic measurements in the laboratory are affected by factors such as scattering attenuation and diffraction effects, which seriously affect the stability and accuracy of the Q value measurement of the sample. The factors of laboratory scattering attenuation and diffraction effects are frequency dependent, the LSR and CFS are not sufficient to obtain the accurate Q value when considering frequency dependence. The Q values estimated from the aluminum sample No.2 and the epoxy resin samples show that the LSR and CFS will underestimate the Q value. The seismic physics model example shows that when the ultrasonic transducer whose main frequency is at least greater than 100 kHz is used in attenuation measurement in the laboratory, Q value is highly dependent on frequency. The TPM is more accurate to estimate the frequency-dependent Q value than LSR and CFS, which can be used as a theoretical reference for measuring the attenuation in the laboratory.

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References


