Source wavefield reconstruction using the Remez exchange algorithm for reverse time migration

Introduction

Reverse time migration (RTM) is a state-of-the-art technique to image complex structures and special geologic targets (McMechan 1983; Zhang & Sun 2009; Liu et al. 2011). The crosscorrelation imaging condition requires the source and receiver wavefields available at the same time step. The source and receiver wavefields are propagated in the forward and reverse time direction, respectively. Hence, the source wavefields should be reconstructed on the fly, and accessed to compute the correlation with the receiver wavefields during the backward propagation. Two practical wavefield reconstruction methods are the checkpointing method and the boundary value method. Griewank & Walther (2000) proposed the checkpointing method, which keeps the wavefields at several time steps (checkpoints) in memory during the forward propagation, and then uses them as the initial conditions to remodel the source wavefields during the backward propagation. Afterwards, Symes (2007) implemented RTM with the optimal checkpointing to make trade-off between storage and computational cost. The checkpointing scheme generally reduces the memory complexity at the cost of increasing the amount of calculation. The boundary value method reconstructs the source wavefields by the wavefields at \( M \) layers of the spatial grid points in the boundary and the snapshots at the final two time steps (Gauthier et al. 1986; Dussaud et al. 2008) for a \((2M+1)\) grid point finite-difference (FD) stencil. The conventional boundary value method is accurate but memory-expensive. To reduce storage overhead, Liu et al. (2015) shown a boundary value method using a linear combination of boundary wavefields to reconstruct the source wavefields. The memory usage is \( 1/M \) of the conventional \( M \)-layer method. However, the accuracy is insufficient because their method is under the least-square (LS) criterion.

On the basis of the one-layer method of Liu et al. (2015), we develop an efficient source wavefield reconstruction method, in which the reconstruction coefficients are optimized by the Remez exchange algorithm. We test the proposed method on synthetic data in 3D RTM.

Method

The one-layer boundary-value method discretizes the spatial derivatives at layers between 0 and \( M-1 \) as (Liu et al. 2015)

\[
\frac{\partial^2 p}{\partial x^2} \bigg|_{x=0} = \frac{1}{h^2} \sum_{i=1}^{M} a_i \left( p_i - 2p_0 + p_{-j} \right),
\]

(1)

\[
\frac{\partial^2 p}{\partial x^2} \bigg|_{x=b} = \frac{1}{h^2} \left[ h \left( p_2 - 2p_0 + p_0 \right) + h^2 \sum_{i=1}^{M} a_i \left( p_{2i} - 2p_i + p_{-j} \right) \right],
\]

(2)

\[
\frac{\partial^2 p}{\partial x^2} \bigg|_{x=(M-1)h} = \frac{1}{h^2} \left[ \sum_{i=1}^{M-1} b_{M-i} \left( p_{M-i} - 2p_{M-i} + p_{M-i} \right) + h^2 \sum_{i=1}^{M} a_i \left( p_{2(M-i)+1} - 2p_{M-i} + p_{-j} \right) \right],
\]

(3)

where, \( M \) is the operator length of FD, \( a_i \) \( (i = 1, 2, \ldots, M) \) and \( b_{j-i} \) \( (j = 1, 2, \ldots, M-1, i = 1, 2, \ldots, j+1) \) denotes the reconstruction coefficients, \( h \) represents the grid spacing. Eqs (1)-(4) can be rewritten as

\[
\frac{\partial^2 p}{\partial x^2} \bigg|_{x=0} = \frac{1}{h^2} \sum_{i=1}^{M} a_i \left( p_i - 2p_0 \right) + A,
\]

(4)

\[
\frac{\partial^2 p}{\partial x^2} \bigg|_{x=b} = \frac{1}{h^2} \sum_{i=1}^{M} b_{j-i} \left( p_{j+i} - 2p_j + p_{j-i} \right) + h^2 \sum_{i=1}^{M} a_i \left( p_{2j-i} - 2p_j + p_{j-i} \right) + b_{j-i} A, \quad A = \sum_{i=1}^{M} a_{j-i},
\]

(5)

\( j = 1, 2, \ldots, M-1 \). The one-layer method only stores a linear combination of boundary wavefields \( A \) during forward propagation and reconstructs the spatial derivatives based on eqs (4) and (5) during backward propagation. The conventional method stores the wavefields at \( M \) layers of \( ih \) \( (i = -1, -2, \ldots, -M) \).

According to the plane-wave theory, we can derive the dispersion relations of eqs (4) and (5)

\[
(k,h)^2 = 2 \sum_{j=1}^{M} a_j \left[ 1 - \cos(ik,h) \right],
\]

(6)
\[
(k, h)^2 \approx 2 \sum_{i=1}^{N} b_{ij} \left[ 1 - \cos(ik, h) \right] + 2 b_{ij+1} \sum_{i=1}^{N} a_{(i+1)j} \left[ 1 - \cos((j+1)k, h) \right], j = 1, 2, \ldots, M-1,
\]

where, \(k_x\) represents the wavenumber along the \(x\)-axis direction. The relative dispersion errors are defined as

\[
\delta_{o}(k, h) = \sum_{i=1}^{M} a_i \phi_i (k, h) - 1,
\]

\[
\delta_{j}(k, h) = \sum_{i=1}^{M} b_{ij} \phi_i (k, h) + b_{ij+1} \sum_{i=1}^{M} a_{(i+1)j} \psi_{j,i} (k, h) - 1, j = 1, 2, \ldots, M-1,
\]

\[
\phi_i (k, h) = \frac{2 \left[ 1 - \cos(ik, h) \right]}{(k, h)^2}, \quad \psi_{j,i} (k, h) = \frac{2 \left[ 1 - \cos((j+1)k, h) \right]}{(k, h)^2}.
\]

We then optimize the reconstruction coefficients based on the minimax approximation. The objective functions is

\[
E_o = \max_{k, h \in [0, \beta_o]} \left| \delta_{o}(k, h) \right|, \quad \beta_o (j = 0, 1, \ldots, M-1)
\]

\[
E_j = \max_{k, h \in [0, \beta_j]} \left| \delta_{j}(k, h) \right|, \quad \beta_j (j = 0, 1, \ldots, M-1)
\]

where, \(\beta_j, (j = 0, 1, \ldots, M-1)\) is the upper limit of \(k, h\). The Remez exchange algorithm is used to solve the minimax problem (Yang et al. 2017; He et al. 2019). According to the minimax criterion, the dispersion errors alternate between the maxima and minima within the wavenumber range. We have

\[
\delta_{o}(k, h) = (-1)^l \lambda_o, l = 1, 2, \ldots, M,
\]

\[
\delta_{j}(k, h) = (-1)^l \lambda_j, j = 1, 2, \ldots, M-1, l = 1, 2, \ldots, j+1.
\]

The steps of optimizing the reconstruction coefficients are:

a. Solve the linear equation (13) to obtain \(a_i\) and \(\lambda_o\).

b. Compute \(E_o\) by equation (11) and compare it and the allowed error \(\eta\). If \(E_o > |\eta|\), go to step c.

c. Search for the extremum points of \(\delta_{o}\).

d. Use the extremum points as the new sampling points and solve the linear equations (13) to get new \(a_i\) and \(\lambda_o\).

e. Repeat steps b–d.

f. Go to step g until the convergence condition \((E_o \leq |\eta|)\) is satisfied.

g. Substitute the optimized \(a_i\) into equation (14) and solve the linear equations to yield \(b_{ij}\) and \(\lambda_j\).

h. Compute \(E_j\) via equation (12) and compare them and \(\eta\). If \(E_j > |\eta|\), go to step i.

i. Search for the extremum points of \(\delta_{j}\).

j. Use the extremum points as the new sampling points and solve the linear equations (14) to give new \(b_{ij}\) and \(\lambda_j\).

k. Repeat steps h–j.

l. If the convergence condition \((E_j \leq |\eta|)\) is satisfied, we stop.

For the given allowed error \(\eta\) and initial \(\beta_j, (j = 0, 1, \ldots, M-1)\), we can obtain the optimal coefficients and \(\beta_j\) according to steps a-l. The effective bandwidth is automatically determined by

\[
\beta_j = \min_{j=0,1,\ldots,M-1} \beta_j.
\]

The minimax approximation ensures that the reconstruction errors do not exceed the allowed limit \(\eta\) within \([0, \beta_j]\).

Figure 1 displays the dispersion curves of different methods. It is seen that the \(M\)-layer method is preferable to two one-layer methods. The proposed one-layer method based on the Remez exchange algorithm is more accurate than the existing one-layer method based on LS.
Figure 1. The dispersion errors of different methods with $M = 6$ and $\eta = 0.001$. (a) The conventional $M$-layer method. (b) The existing one-layer method. (c) The proposed one-layer method.

Examples

We use the salt model to test the proposed method. The grid dimensions are $338 \times 338 \times 210$, the grid spacing is 12 m, the time step is 1.0 ms, and the recording time is 3.0 s. A Ricker wavelet with a peak frequency at 25 Hz is used as a source in modelling and RTM. 100 sources and 10000 receivers are evenly distributed on the surface. The source and receiver intervals are 360 and 10 m, respectively. Figure 2 illustrates the images of RTM in the slices of $x = 2040$ m, $y = 2040$ m and $z = 960$ m with different reconstruction methods. For details, Figure 3 shows the difference between the images of the $M$-layer method and other two one-layer methods. We can observe that the error of the images of the proposed method is much smaller than the error of the existing one-layer method.

Figure 3. The images of RTM in the slices of $x = 2040$ m, $y = 2040$ m and $z = 960$ m with different methods for the salt model. (a) The existing one-layer method. (b) The proposed one-layer method.
Figure 4. The difference between the images of the $M$-layer method and the one-layer methods. (a) The existing one-layer method. (b) The proposed one-layer method.

**Conclusions**

We develop a new source wavefield reconstruction method, which reconstructs the source wavefields using a linear combination of boundary wavefields. We optimize the reconstruction coefficients based on the minimax approximation with the Remez exchange algorithm. The proposed method has higher accuracy than the one-layer method based on LS. The proposed method can obtain acceptable images, which are close to the images of the conventional method storing $M$ layers of boundary wavefields. The memory usage of the proposed method is just $1/M$ of the $M$-layer method.

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**References**


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