The estimation of permeability using grain size distribution and pore space analysis

Introduction

Fluid flow analysis in reservoirs considerably influences the efficient production of underground resources such as hydrocarbons (oil and gas). The permeability of reservoir rock is one of the essential parameters for estimating the efficiency of production. Permeability defines the transport properties of the fluid and has a significant impact on the production efficiency in underground resource development. The permeability of porous media is often estimated using empirical equations (Carman, 1937). However, the relationship between the grain size distribution of reservoir rock and permeability is still left in the blank. The grain size distribution of sandstone follows the Weibull distribution to some extent (Esmaeelnejad et al., 2016), and the distribution is determined by the shape parameter. The accuracy of fluid flow analysis in reservoirs could be improved remarkably as long as some relationship between the permeability, and the grain size distribution parameters can be specified appropriately.

In this study, several porous rock models with different shape parameters are created, and fluid flow simulations are performed using the Smoothed Particle Hydrodynamics (SPH) method (Lucy, 1977; Muller et al., 2003), which is a particle method. The variation of permeability is investigated as a function of the shape parameter. The change in permeability due to the shape parameter is investigated accordingly. Besides, we analyze characteristics of the pore space of digital specimens (Mehlhom et al., 2008) to obtain the cross-sectional area projected in the flow direction of the poised throat, and propose an approximate equation estimate the permeability in the reservoir rock from these data.

Method

SPH method belongs to the category of particle methods. In this method, objects are regarded as an aggregate of particles. The physical quantity $\phi$ of each particle is calculated by equation (1)

$$\phi(r) = \sum_b m_b \frac{\phi_b}{\rho_b} W(r - r_b, h).$$

In this equation, $r$ is the position vector of the particle, $h$ is the radius of the influence domain, $W$ is the kernel function, $\rho$ and $m$ are the density and mass, respectively, subscript $b$ indicates the neighbouring particles in the influence domain. All kernel functions satisfy equation (2) as below.

$$\lim_{h \to 0} W(r - r_b) = 1.$$  

In the present study, we assume that the grain size distribution is based on the Weibull distribution, as shown in equation (3).

$$F(x) = 1 - \exp \left\{ - \left( \frac{x}{\eta} \right)^m \right\},$$

where $x$ is a stochastic variable, $\eta$ is the position parameter, and $m$ is the shape parameter. The Weibull distribution depends on the shape parameter and the mean value.

When analyzing the pore-throat of a porous rock model, the target of the calculation is considered to be binary data of solid and liquid. The coordinates located at the same distance from the four solid particles are defined as the pore centres. The intermediate axis between the three solid particles is defined as the pore-throat illustrated in Figure 1.

The cross-sectional area of the pore-throat between the centres of the pores is calculated using equation (4).
Figure 1 An example of models of porous media. Red dots are pore centers, green dots are pore throats, and blue spheres are solid particles.

\[ S = \sqrt{(a + b + c) abc} - \pi \left( \frac{a^2 \theta_1}{360} + \frac{b^2 \theta_2}{360} + \frac{c^2 \theta_3}{360} \right) \]  

(4)

In this equation, a, b, and c are the radii of the three solid particles in contact with each other at the pore throat. \( \theta_1 \), \( \theta_2 \), and \( \theta_3 \) are the angles of the triangles composed of three solid particle centres. The cross-sectional area \( S_2 \) projected in the flow direction is expressed in equation (5) as follows.

\[ S_2 = S \times \frac{|x(i)-x(j)|}{\sqrt{(x(i)-x(j))^2+(y(i)-y(j))^2+(z(i)-z(j))^2}} \]  

or \[ \frac{|y(i)-y(j)|}{\sqrt{(x(i)-x(j))^2+(y(i)-y(j))^2+(z(i)-z(j))^2}} \]  

(5)

\( x, y, z \) are coordinates of the connected pore centres.

Numerical models

Figure 2 An example of digital rock specimens. Yellow is solid particles. Fluid flow between gaps of yellow particles.

Table 1 Parameters of models of porous media.

<table>
<thead>
<tr>
<th>Model</th>
<th>Average grain size(m)</th>
<th>Shape parameter</th>
<th>Model size(m)</th>
<th>Porosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model1</td>
<td>1.2 \times 10^{-4}</td>
<td>1</td>
<td>6.5 \times 10^{-4} \times 6.5 \times 10^{-4} \times 6.5 \times 10^{-4}</td>
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</tr>
<tr>
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<td>5.2 \times 10^{-4} \times 5.2 \times 10^{-4} \times 5.2 \times 10^{-4}</td>
<td>0.165</td>
</tr>
<tr>
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<td>0.179</td>
</tr>
<tr>
<td>Model4</td>
<td>1.2 \times 10^{-4}</td>
<td>3</td>
<td>4.25 \times 10^{-4} \times 4.25 \times 10^{-4} \times 4.25 \times 10^{-4}</td>
<td>0.179</td>
</tr>
<tr>
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<td>4.12 \times 10^{-4} \times 4.12 \times 10^{-4} \times 4.12 \times 10^{-4}</td>
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<td>0.172</td>
</tr>
<tr>
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<td>4.0 \times 10^{-4} \times 4.0 \times 10^{-4} \times 4.0 \times 10^{-4}</td>
<td>0.174</td>
</tr>
<tr>
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<td>5</td>
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<td>0.172</td>
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<td>0.178</td>
</tr>
</tbody>
</table>
A numerical porous media model is introduced shown in Figure 2 to illustrate our proposed method. This porous media model is created using 10 different shape factors where the model parameters are defined in Table 1. The number of solid particles in the model is set to 50, and periodic boundaries are set in all directions. Since the arrangement of the solid particles also affects the permeability, 10 patterns of particle arrangement are created, and the average permeability is measured accordingly. In the numerical test, the fluid flow direction is set to x-, y-, and z-directions.

Results

![Figure 3 Cross-plot of shape parameter and permeability. Red dots represent the permeability of each realization. Blue dots are its averaged values.](image)

The cross-plot of the shape parameter and permeability is shown in Figure 3. The red dots represent the numerical results for each particle configuration, and the blue ones indicate the average. It is observed from the Figure that the shape parameter has a strong influence on the permeability in a linear manner.

![Figure 4 Cross-plot of shape parameter and cross-sectional area of pore throat.](image)

Similarly, Figure 4 shows a cross plot of the shape parameter and the cross-sectional area of the pore throat projected in the flow direction. With the same tendency shown in Figure 3, the cross-sectional area decreases when the shape parameter is increasing.
Estimate equation

The approximation form is indispensable to ease the practical implementation. Based on the data obtained from the numerical experiments, the approximate equation (6) is obtained by the nonlinear least-squares method.

\[ P = 0.0538x + 5.82 \times 10^{-10} e^{-2.69y} + 4.0 \times 10^{-11}. \]  

(6)

where \( P \) represents the permeability, \( x \) is the cross-sectional area of the pore throat, and \( y \) is the shape parameter. The cross-plot of the shape parameter and the permeability from the simulated and the approximated results is shown in Figure 5, where the coefficient of determination is 0.87, indicates that the approximation can well represent the simulation results. We can accurately estimate the permeability for the fluid analysis with the information of the cross-sectional area and the shape parameter.

Conclusion

![The cross-plot of the shape parameter and the permeability from the simulated and the approximated results](image)

\textit{Figure 5 The cross-plot of the shape parameter and the permeability from the simulated and the approximated results}

We investigate the relationship between the grain size distribution and the permeability and the average cross-sectional area of pore throats projected in the flow direction. In the numerical experiments with our proposed method, the porous rock model is introduced and tested. An approximate equation for determining the permeability is developed accordingly based on the obtained data. The estimated equation shows a high agreement with the numerical results. It is indicated that this expression could be useful for assessing the permeability from the geometric properties of the reservoir rock.

References


