Simultaneous up-down separation and Vz denoise using joint sparsity recovery

Introduction

Ocean bottom geophone data are often contaminated by shear-like noise (“Vz-noise”) caused by shot energy scattering off sea-floor heterogeneities and propagating as interface waves (Paffenholz et al., 2006b). Although this noise is weak or absent on hydrophone recordings, it is often strong enough on the geophone recordings that it requires removal before either up-down wavefield separation or PZ summation can take place (e.g., Craft and Paffenholz, 2007; Hampson and Szumski, 2020). The strength of the noise depends on the ocean bottom conditions and its propagation velocity is extremely low at around 90% of the shear velocity. As a result, Vz-noise is severely aliased and apparently unpredictable, even between geophones spaced only a few metres apart.

Here, we apply a distributed compressive sensing approach (Baron et al., 2009) called joint sparsity recovery to the geophone and hydrophone data to perform up-down wavefield separation and isolation of the Vz-noise. This work was motivated by the work of Oghenekohwo et al. (2017) and Tian et al. (2018) from which we recognised that the problem of up-down wavefield separation bore many similarities to the noisy time-lapse estimation problem dealt with by those authors.

Method

The total pressure field can be represented as the superposition of the up- and down-going wavefields,

\[ P = D + U. \]  (1)

If we assume that any wavelet or scaling differences have been removed, use of the equation of motion allows us to deduce that the up- and down-going components of the pressure field can be derived from a linear combination of the pressure and particle velocity measurements,

\[ U = \frac{1}{2} \left( P - \frac{\rho \alpha}{\cos \theta} V_z \right), \quad D = \frac{1}{2} \left( P + \frac{\rho \alpha}{\cos \theta} V_z \right). \]  (2)

Let the pressure-normalised particle velocity be,

\[ Z = \frac{\rho \alpha}{\cos \theta} V_z. \]  (3)

Substitution of (3) into (2), solving for \( Z \) and then adding a term for Vz-noise we find that,

\[ Z = D - U + \lambda N. \]  (4)

Inspired by the joint sparsity recovery approach, we write the unknown and recorded fields as column vectors and combine (1) and (4) in the system of equations,

\[ \begin{bmatrix} I & I & 0 \\ I & -I & \lambda I \end{bmatrix} \begin{bmatrix} d \\ u \\ n \end{bmatrix} = \begin{bmatrix} p \\ z \end{bmatrix}, \]  (5)

which expresses the pressure-normalised data \((p, z)\) as simple linear combinations of the up-going \((u)\) and down-going \((d)\) wavefields along with additive Vz-noise \((n)\). Although we have used identity matrices, there is latitude to use other operators. The system of equations is under-determined and so cannot be solved uniquely without some form of regularisation. The distributed compressive sensing approach of Baron et al. (2009) describes a forward model in which all measurements share a common sparse component while each individual measurement contains an innovation component in a sparsity...
promoting transform domain. A somewhat similar problem exists in the differencing of 4D snapshots, which are assumed to have some common features, some unique features and additive noise. Oghenekohwo et al. (2017) took an interesting approach which has since also been used in a slightly different manner by Tian et al. (2018) to attenuate the incoherent noise while taking 4D differences.

Let us assume that the known and unknown vectors are already represented in a sparsity promoting transform domain. Furthermore let,

$$
A = \begin{pmatrix}
I & I & 0 \\
I & -I & \lambda I
\end{pmatrix}, \quad \begin{pmatrix}
x \\
u \\
n
\end{pmatrix} = \begin{pmatrix}
d \\
p \\
z
\end{pmatrix}.
$$

(6)

Using these definitions, we may write the solution of (5) in the form of a basis pursuit problem,

$$
\arg\min_{x} \mu \frac{1}{2} \|x\|^2 + \frac{1}{2} \|Ax - b\|^2, \quad \text{st} \quad Ax = b,
$$

(7)
in which \( \mu \) is a trade-off parameter controlling the importance of the L1 versus L2 norm. Since the solutions are in the (sparsity promoting) transform domain, inverse transformation is required to complete the procedure.

The choice of sparsity promoting transform is an important consideration. We found that although several transforms produced good results, some approaches were significantly more efficient than others. However, it should be noted that we only need to forward transform of the input data and inverse transform the solutions once.

Examples

We first show a synthetic example to demonstrate the performance of the proposed approach. We used a similar elastic model to Paffenholz et al. (2006a) to recreate the pressure and the particle velocity wavefield. The model is shown in Figure 1.

![Figure 1 - Elastic finite-difference model with many scatterers at the seabed. The shallowest seabed layer is 3 m thick with a shear velocity of 115 ms⁻¹. The shear speed of the scatterers is perturbed randomly in the range of 0-25 % of the seabed shear velocity](image)

The modelled wavefields, which are shown in Figure 2, were created using a 2D elastic finite difference code (Thorbecke and Draganov, 2011). It is evident that the Vz-noise is weak or absent on the hydrophone data but clearly present on the geophone data. The noise exhibits the expected characteristics, i.e., coherency on the common-receiver gathers (Figure 2b) and apparent incoherency on the common-shot gathers (Figure 2d). We applied our proposed approach (7) to the modelled data in Figure 2. The results are shown in Figure 3 in the common-shot and common-receiver domains. The
noise model is recovered well although there is some weak leakage at ~1.4 s (indicated by white arrows).

**Figure 2** - Common-receiver and common-shot gathers for pressure and geophone data using the model in Figure 1. Scattered energy is coherent in the common-receiver domain and incoherent in the common-shot domain.

At this location, we notice that the hydrophone data is weaker than the geophone data because the first order water bottom multiple and a deeper reflection ($z = 1260$ m) to have arrived at about the same time. As a result, destructive interference occurs in the hydrophone data, whereas constructive interference takes place in the geophone data. The leakage is mild, easily ameliorated and must be balanced against the obvious efficacy of the noise estimate. We conclude that our proposed approach can produce good estimates of the Vz-noise field.

**Figure 3** - Results showing the noise free particle velocity data and the Vz-noise model in both common-shot and common-receiver domains. The denoised geophone data is much more similar in character to the hydrophone data which was shown in Figure 2a and Figure 2c.

Our second example is real OBN data from the North Sea. Figure 4a shows the input geophone data in the common-receiver domain containing a significant amount of Vz-noise. Figure 4b and Figure 4c show the denoised data and the estimated Vz-noise using the proposed approach. The Vz-noise has been very well estimated and removed from the geophone data. As a result, the particle velocity data is much more comparable in amplitude and character to the pressure data (not shown), which should be expected for effective wavefield separation.
Figure 4 - A common-receiver gather from an OBN survey acquired in the North Sea: a) input geophone data; b) noise attenuated output; c) estimated Vz-noise. (Data courtesy of AGS and TGS)

Conclusions

A new method of estimating Vz-noise and performing up-down separation simultaneously has been presented. The method is based on distributed compressive sensing, in which jointly-sparse signals are recovered from a simple set of linear equations. The synthetic and field data example demonstrates the effectiveness of the proposed approach.

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References

Hampson, G. and G. Szumski, 2020, Down/down deconvolution: 82nd Annual International Conference and Exhibition, EAGE, Extended Abstracts, E104