3D transdimensional ambient noise surface wave tomography of the Reykjanes Peninsula – a feasibility study

Introduction

Seismic surface-wave tomography is a well-known and popular method to obtain the Earth's surface wave velocity structure. Many different techniques and algorithms have been proposed and tested. Conventional linearized or gradient base iterative inversion schemes usually do not include a detailed assessment of the uncertainty (Young et al., 2013). In addition, such schemes require an (a-priori) prescribed cell size, which does not account for spatial variations in sampling (i.e., a non-uniform ray coverage). The transdimensional hierarchical Bayesian method introduced by Bodin et al. (2012) addresses these two limitations. The transdimensional method is a Bayesian inference method relying on reversible jump Markov chain Monte Carlo (rjMcMC) and Voronoi cells to generate samples from the posterior probability distribution of the model given the observed data. Young et al. (2013) and Galetti et al. (2017) used a two-step transdimensional scheme to recover the 3D surface wave velocity structure. The first step involves the recovery of 2D maps of phase velocity using frequency-dependent travel time data employing the 2D transdimensional tomography method of Bodin et al. (2012). This results in laterally varying dispersion curves (phase velocities that are frequency-dependent). These dispersion curves are subsequently inverted separately using a 1D transdimensional approach, which results in a collection of spatially varying 1D velocity models. The estimated depth-dependent velocity models are finally interpolated to build a 3D velocity structure of the subsurface.

Zhang et al. (2018) showed that the two-step inversion scheme could be biased and may cause loss of information. Hence, they proposed a 3D transdimensional approach that uses a 3D discretization of the subsurface using a Voronoi polyhedral tessellation. This method has a comparable computational cost as the two-step transdimensional method but preserves valuable information and results in a more accurate velocity structure and a better interpretable uncertainty. In this study, we investigate the feasibility of this 3D inversion method (Zhang et al., 2018; Zhang et al., 2020) to recover 3D velocity structure using ambient noise data recorded over the Reykjanes Peninsula (Weemstra et al., 2021). To that end, we have generated synthetic travel time data using an array of stations over the Reykjanes Peninsula (henceforth referred to as the RARR; Weemstra et al., 2021). Specifically, we prescribed a 3D checkerboard velocity distribution. The frequency-dependent travel times, which can be derived from phase velocity data, are then used as input for the 3D transdimensional algorithm. We find that the algorithm works well for the RARR with the reliable frequency range of data.

3D transdimensional surface wave tomography

Transdimensional tomography is a Bayesian inference method that uses Voronoi cells in conjunction with a reversible jump Markov chain Monte Carlo (rjMcMC) algorithm. The reverse jumps allow for a variable number of Voronoi cells, and hence a variable number of parameters. The algorithm involves different types of model perturbations. Specifically, we use four different perturbation types to efficiently sample the posterior distribution (Bodin and Sambrige, 2009): velocity update, Voronoi cell move, death, and birth. In addition, we perturb the amplitude of the noise to infer the posterior probability of the errors on the measured surface wave travel times (This is introduced by Bodin et al., 2012). These perturbation types allow the model to dynamically adapt to both data density, underlying velocity structure, and travel time noise. At each step of the Markov chain, a new sample is drawn by perturbing the 3D velocity structure (using one of the four perturbation types). The surface wave dispersion data (i.e., the frequency-dependent travel times) are then calculated to evaluate the following acceptance probability (Bodin and Sambrige, 2009):

\[
\alpha(m'|m) = \min \left[ 1, \frac{p(m')}{p(m)} \times \frac{p(d_{obs}|m')}{p(d_{obs}|m)} \times \frac{q(m|m')}{q(m'|m)} \times |J| \right],
\]

where \(\alpha(m'|m)\) is the probability of accepting the proposed model \(m'\) given the current model \(m\), \(\frac{p(m')}{p(m)}\) the prior probability ratio of two models, \(d_{obs}\) the observed data (here these are the frequency-dependent travel times), \(\frac{p(d_{obs}|m')}{p(d_{obs}|m)}\) the likelihood ratio of the two models, \(\frac{q(m|m')}{q(m'|m)}\) the proposal ratio, and \(J\) is the Jacobian matrix, which accounts for (potential) differences in dimensionality between \(m\) and \(m'\) (resulting from a different number of Voronoi cells).
When sufficient samples are drawn from the posterior, we can compute mean, variance, or other statistical measures of the posterior. The first samples of the chain are discarded. This initial sampling is usually referred to as the burn-in period, which is the period that the algorithm needs to reach sufficient mixing of the posterior samples. Since each sample is drawn based on a small perturbation of its previous model, adjacent samples are correlated or similar to each other. To ensure that the drawn samples are uncorrelated, we retain samples every so many some iterations; for example every 250th sample is retained. This process is usually referred to as ‘thinning’.

The dependence of the posterior on the data is encoded in the likelihood function $p(d_{obs}|m)$. To evaluate the likelihood of the proposed model given the observed data, and compare this likelihood with the likelihood of the current model in the chain, we need to compute the frequency-dependent travel times in the proposed model. This is achieved by employing the two-step approach detailed in Zhang et al. (2018). First, at each point on the Earth’s surface, the local frequency-dependent dispersion curve is computed. Then, these frequency-dependent dispersion curves are computed using a modal approximation method (Herrmann 2013). As such, we obtain frequency-dependent maps of surface wave velocity. In the second step, we use these maps to compute frequency-dependent travel times using the fast-marching method (Rawlinson & Sambridge 2004).

The transdimensional tomographic method is very flexible when it comes to combining different data scales or different data types. It is self-regularized and self-smooth and hence doesn't require any regularization or smoothing. In addition, model uncertainty is naturally captured in the posterior. Despite these benefits, the high computational cost is still a drawback of the method. Computing frequency-dependent travel times using the fast-marching method contributes most to the computational cost. Once the ray paths are determined, integration of the slowness along each ray path is straightforward and relatively fast. To make the algorithm computationally less demanding, Bodin et al. (2012) updated the ray paths only occasionally (three times, every one million samples). In this way, they partially linearized the algorithm. We propose to update ray paths at the same level of thinning as ray paths do not change too much in thinning periods. In this way, we reduce the computational cost significantly while still retaining most of the non-linearity.

Checkerboard tests for the RARR

The RARR consists of 83 seismic stations (depicted in Figure 1(a) by triangles) recording passive seismic noise in Reykjanes Peninsula in southwest Iceland. As it can be seen, the distribution of stations is non-uniform, i.e., the station coverage is dense in one area while it is sparse in other areas. A non-uniform distribution of stations implies that the achievable phase-velocity resolution can be expected to vary significantly across the region covered by the seismic array, higher in the regions that are more densely covered by stations and decreasing where station density is low. To investigate the 3D transdimensional method proposed by Zhang et al. (2020), we designed a synthetic model for the station configuration of the RARR. Figure 1(b) shows a checkerboard velocity model for the region. The size of the checkers is $10 \times 10 \times 5$ km. The checkerboard consists of checkers with two distinct velocity profiles shown in Figure 2(a). The middle panel of Figure 2 depicts the sensitivity kernels of surface wave (Herrmann 2013). It shows that the longer periods are mostly sensitive to the velocity at depths below 5 km, whereas the shorter periods have greater sensitivity to the velocity at shallower depths. Using surface wave travel times at these frequencies, we therefore expect the algorithm to resolve both lateral and vertical velocity structure.

![Figure 1](image-url)  
*Figure 1. (a) Geographical locations of the stations of the RARR. (b) 3D synthetic checkerboard model to investigate transdimensional method. The second layer is extended infinitely in the depth direction.*
Figure 2. Analyzing two distinct depth profiles of the assumed checkerboard. (a) depth profile surface wave velocity (left), the sensitivity of phase velocity to the s-wave velocity variation (middle) for different time periods (with different colors), and phase velocity dispersion (right). (b) same as (a) for the other checker.

The reliable range for picking the phase velocity in the field data (time-averaged cross-correlations of ambient seismic noise) is 0.1 to 0.5 Hz. Hence, we calculated phase velocities for all the receiver pairs in this frequency range. We used a grid size with a spacing of 0.5 km to compute synthetic data. Subsequently, these travel times are used to test the transdimensional algorithm. The initial velocity model, the number of cells, and the noise parameters are all generated randomly within each parameter's prior range. We used grid sizes with a spacing of 1 km in the depth direction and 2 km in horizontal directions to compute travel times using the fast-marching method at each iteration of the McMC algorithm. We used ten parallel chains of McMC and generated one million samples at each chain. Samples have been retained at every 250 iterations to make sure there is no correlation between samples. Ray paths have been updated at every 250 iterations, which reduces computational costs significantly by making the algorithm 20 times faster than updating ray paths at each iteration. The first 4e5 samples have been discarded as the burn-in period. Hence, we have 2400 samples per chain, in total, 24000 samples are used in the computation of average model and uncertainty (i.e., standard deviation of accepted samples).

Figure 3 demonstrates the ability of the 3D transdimensional algorithm to recover the checkerboard in Figure 1(b) in a noise-free experiment (b-c) as well as an experiment with a relatively high level of additive random noise (d-e). Figure 3(a) shows the actual surface wave velocity in three different 2D profiles. Figures 3(b, d) show the post-burn-in average of sampled surface wave velocity models. As you can see, there is a good match between the actual velocity model and the recovered average models, especially in the first vertical checker. As we expected (based on Figure 2), the resolutions of recovered models in deeper checkers are less because of the lower sensitivity of surface wave kernels. In the experiment with the additive noise level, the recovered model is smoother than the experiment without additive noise that means the algorithm adapted model resolution to the noise. The algorithm provides us with an estimate of uncertainty depicted in Figure 3(c, e) which is the variance of post-burn-in sampled models of the posterior. Higher uncertainty is visible for layer interfaces, areas with low coverage of ray density, and deeper parts of the model.

Conclusions

In this paper, we investigated 3D transdimensional tomography to recover the 3D surface wave velocity structure of Reykjanes area. We proposed the ray paths update step instead of updating ray paths at all iterations of transdimensional McMC, which led to a significant reduction of computational cost while still accounting for the non-linearity of the problem sufficiently. Our results show that the algorithm successfully adapted to the rays' density by yielding a higher resolution and smaller uncertainty in areas with a higher density of rays. In contrast, the uncertainty is larger in regions where the station density is lower. In addition, the algorithm successfully adapts to the input data's noise level that provides higher smoothness of the results by increasing the level of additive random noise. The uncertainty maps aid the interpretability of the results.
Figure 3. a) The true checkerboard model including station locations (left), vertical depth profile at easting of 405km (middle), and vertical depth profile at northing of 7095km (right). Locations of vertical depth profiles are depicted by black dash-line on the top view map. Mean velocity and standard deviation calculated from post burn-in samples in a noise-free experiment (b-c) and a test with additive random noise (d-e).

References