DETERMINING DIMORPHOS' MASS FROM THE WOBBLE OF DIDYMOS

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ABSTRACT

In order to demonstrate the feasibility of the deflection of an Earth threatening asteroid, the DART mission impacted Dimorphos, the asteroid moonlet of Didymos, on 26 September 2022. For the assessment of the efficiency of the momentum transfer, the mass of Dimorphos needs to be known. Late 2026 or early 2027, the Hera mission will reach the Didymos binary system and inspect closely its properties and the consequences of the DART impact. The mass of Dimorphos is so small that it will only marginally be possible to measure its gravitational impact by tracking of the spacecraft trajectory. However, images taken by the Asteroid Framing Cameras aboard Hera will make it possible to measure the wobble of Didymos around the common barycenter introduced by the gravity of Dimorphos. This will enable the estimation of Dimorphos' mass.
In previous work, we simulated observations under different assumptions and estimated the respective accuracies of the wobble reconstruction. Hereing, we focus on the actual estimation of the S/C trajectory relative to Didymos, which comprises the wobble.

1 INTRODUCTION

1.1 DART, Hera, momentum transfer efficiency and the mass of Dimorphos

Planetary defence is the protection of earth from the impact of asteroids. With recent advances in technology, the deflection of an hazardous asteroid is now becoming possible.
On 26 September 2022, NASA's Double Asteroid Redirection Test (DART) mission impacted Dimorphos, the moon of near-earth asteroid (65803) Dimorphos, performing the world's first planetary defence test [4]. It reduced the ≈ 12 hours orbital period of Dimorphos around Didymos by 33 minutes [5], corresponding to a velocity change of 2.7 ± 0.1 mm/s [3].
The momentum transfer from the DART spacecraft to Dimorphos is enhanced by the reaction force from the motion of the impact ejecta, as they move mostly in the direction of the impacting spacecraft. To understand the deflection process and to apply the result of the DART impact test to other asteroids, it is required to determine the momentum transfer efficiency β, defined as the ratio between the momentum change of Dimorphos and the momentum of the incoming DART spacecraft. As the mass and velocity of DART and the velocity change of Dimorphos due to the impact are known, the only unknown in the determination of β is the mass of Dimorphos.
The measurement of the mass of Dimorphos is one of the goals of ESA's Hera mission [2]. It will be launched in October 2024 and rendezvous with the Didymos system end of 2026 or beginning of

2027. As the ≈ 160 m sized Dimorphos orbits Didymos (≈ 780 m diameter) at a distance of only 1.2 km, the gravity field in the system is dominated by Didymos and it is challenging to determine Dimorphos' mass through the acceleration of the Hera spacecraft by the gravity of the two asteroids. Therefore, a different method is foreseen to measure the mass of Dimorphos to the required accuracy of 10 %: measuring the wobble of Didymos due to the orbital motion of Dimorphos.

1.2 The wobble

The wobble of Didymos is illustrated in Fig. 1. In a previous study [1], we had investigated the accuracy which can be achieved in the determination of the secondary's mass by measuring the primary's wobble. This comprised basically three steps:

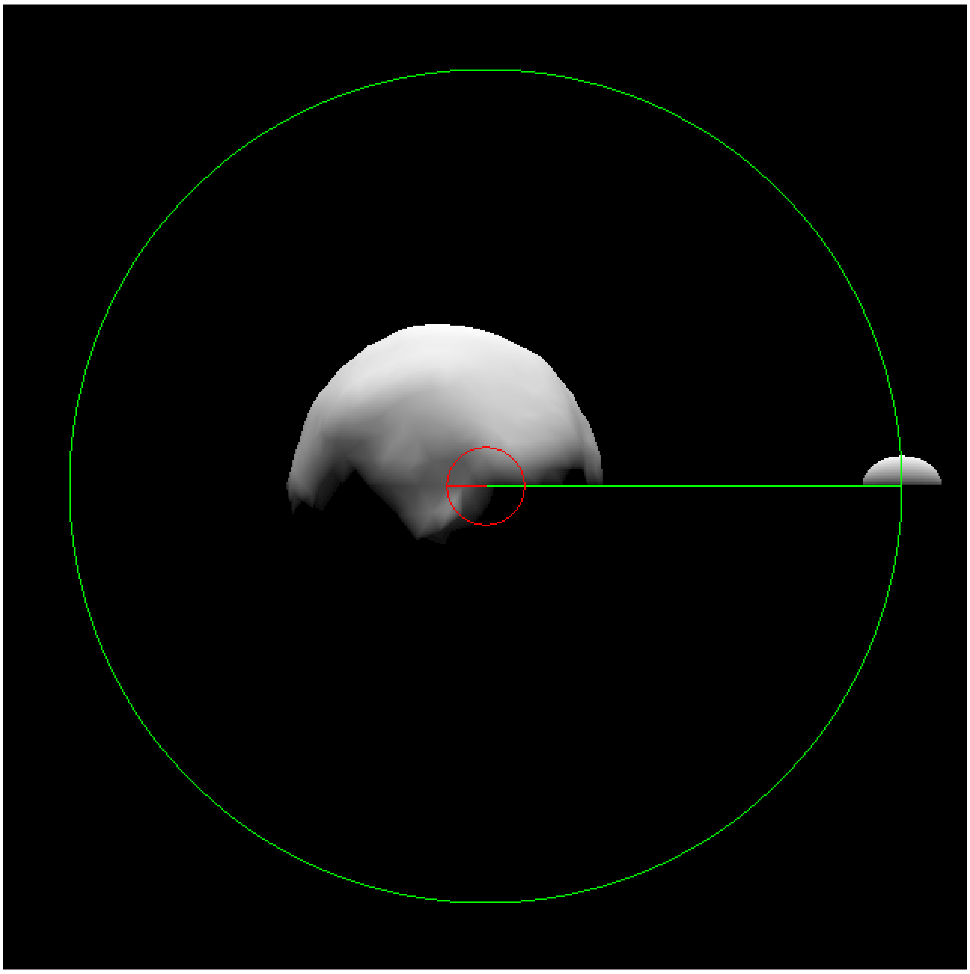

Figure 1: Illustration of the wobble: Didymos and Dimorphos both orbit their common barycenter (red and green orbit, respectively). The ratio of their orbit radii depend on their mass ratio.

1. Generating ensembles of simulated observations under different assumptions.
2. Reconstructing from these simulations the S/C trajectory relative do Didymos (which comprised the wobble).
3. Extracting the radius of the wobble from the reconstructed trajectory.

Herein, we shall elaborate on step 2 and describe in detail the iterative reconstruction of the S/C trajectory, landmark positions in a body fixed frame, and orientations of Didymos. We shall not discuss the extractions of the wobble from the reconstructed trajectory. This is rather simple and various methods are possible. It may just be worth noting that the period of the wobble and its direction at any time are know and that this facilitates its separation from the inertial S/C movement.

2 DATA

The data comprising the required information are apparent landmark directions in images, see section 2.1. If we knew the orientations of Didymos at the image acquisition times, this would be sufficient to achieve convergence of the iteration from arbitrary initial conditions. However, we assume that we do *not* know the asteroid orientations. In this case, the initial state of some variables need to be approximately known, see sections 2.2 and 2.3.

2.1 Apparent landmark directions in images

We assume we have n images taken by the Asteroid Framing Camera (AFC) aboard Hera and m landmarks on the surface of Didymos. Herein, we do not discuss the process of actually defining landmarks and measuring their pixel positions in images. From the Rosetta mission we know that this can be done with about one pixel accuracy.
The S/C has star trackers which provide the absolute orientation in an inertial frame. Therefore, if a landmark j is visible and illuminated in image i, we know the direction vector $\mathbf{v}_{i,j}$ from the S/C to the landmark in an inertial frame.

2.2 Approximate asteroid orientations at image acquisition times

In order to ensure convergence of the iterative solution procedure, we need approximate initial asteroid orientations in each image. Let $R_i^{(k)}$ be the rotation matrix describing the orientation of Didymos in image i at iteration k. Then $R_i^{(1)}$ need to be know approximately. These need not to be very accurate. They can be off a few tens of degrees.

2.3 Approximate S/C positions at image acquisition times

We need also approximate S/C positions relative to Didymos in an inertial frame at image acquisition times. Let $\mathbf{s}_i^{(k)}$ be the S/C position at the acquisition time of image i in iteration k. Then $\mathbf{s}_i^{(1)}$ need to be known approximately. Like the initial asteroid orientations, these need not to be very accurate. The error in the position vectors can be some tens of percent.

3 METHOD

3.1 Approach

We find the solution by iteration. In each iteration k, we estimate simultaneously

- landmark positions $\mathbf{r}_j^{(k)}$ in some body fixed frame,
- asteroid orientations given by rotation matrices $R_i^{(k)}$ at image acquisition times,
- spacecraft positions $\mathbf{s}_i^{(k)}$ relative to the asteroid in an inertial frame at image acquisition times.

3.2 Setup

Let $\mathbf{r}_j^{(k)}$ be the position of landmark j in iteration k in a body fixed frame. Landmark positions are expressed relative to the center of all landmarks. The iteration starts with all landmark positions set to zero, thus

$$\mathbf{r}_j^{(1)} = (0,0,0) \tag{1}$$

for all $j = 1, \ldots, m$.
The number of iterations we use is set to $l = 10^6$. This is quite a generous value. It could probably be smaller, but the computation time was acceptable (in the order of hours on a laptop computer) and the large value ensured reliable convergence.
We apply a scaling factor for the step size of the iterations. During the first half of the iterations, this scaling factor is just one, then it is gradually decreased to zero at the end of the iterations. With $k = 1, \ldots, l$ being the iteration count, it is $\gamma^{(k)} = 1$ for $k \le l/2$ and

$$\gamma^{(k)} = \left(\frac{l-k}{l/2}\right)^2 \tag{2}$$

for $k > l/2$. This damping towards the final iterations prevents oscillations of subsequent solutions and enforces convergence; however, the problem at hand is rather well behaved and would not necessarily require such damping.

3.3 Iteration

We compute values for iteration k from values of the previous iteration $k-1$.
The landmark positions $\mathbf{r}_j^{(k-1)}$ are given in a body fixed frame. Their position in the inertial frame at the acquisition time of image i is obtained by applying the respective rotation matrix, thus the rotated positions are

$$\tilde{\mathbf{r}}_{i,j}^{(k-1)} = R_i^{(k-1)} \mathbf{r}_j^{(k-1)}. \tag{3}$$

For each image, we only consider valid landmarks, i. e., those which are visible and illuminated. Let

$$M_i = \{j \mid j \in \{1, \ldots, m\} \wedge j \text{ is valid in image } i\} \tag{4}$$

be the set of indices of landmarks which are valid in image i. For each such (rotated) landmark position $\tilde{\mathbf{r}}_{i,j}^{(k-1)}$, we get the the nearest point $\mathbf{p}_{i,j}^{(k-1)}$ on the line of sight from the S/C position $\mathbf{s}_i^{(k-1)}$ along the observed direction $\mathbf{v}_{i,j}$ of the landmark. This is done using the SPICE [7] API nplnpt. The directions $\mathbf{v}_{i,j}$ are fixed by observation. In order to reconcile these with the landmark positions, we will pull the landmark towards the line of sight and the S/C in the opposite direction, see Fig. 2.
The difference vector from the landmark position to the found nearest point on the line of sight is

$$\mathbf{d}_{i,j}^{(k-1)} = \mathbf{p}_{i,j}^{(k-1)} - \tilde{\mathbf{r}}_{i,j}^{(k-1)}. \tag{5}$$

In order to track the progress of the iterations, we compute the Root Mean Square (RMS) of these distances:

$$\varepsilon^{(k)} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{m_{\text{val},i}}\sum_{j\in M_i}\|\mathbf{d}_{i,j}^{(k-1)}\|^2}, \tag{6}$$

with $m_{\text{val},i} = |M_i|$, cf. Eq. 4. The value of $\varepsilon^{(k)}$ should smoothly go down to zero for $k \to l$ and alright be small compared to its initial value at $k = l/2$.

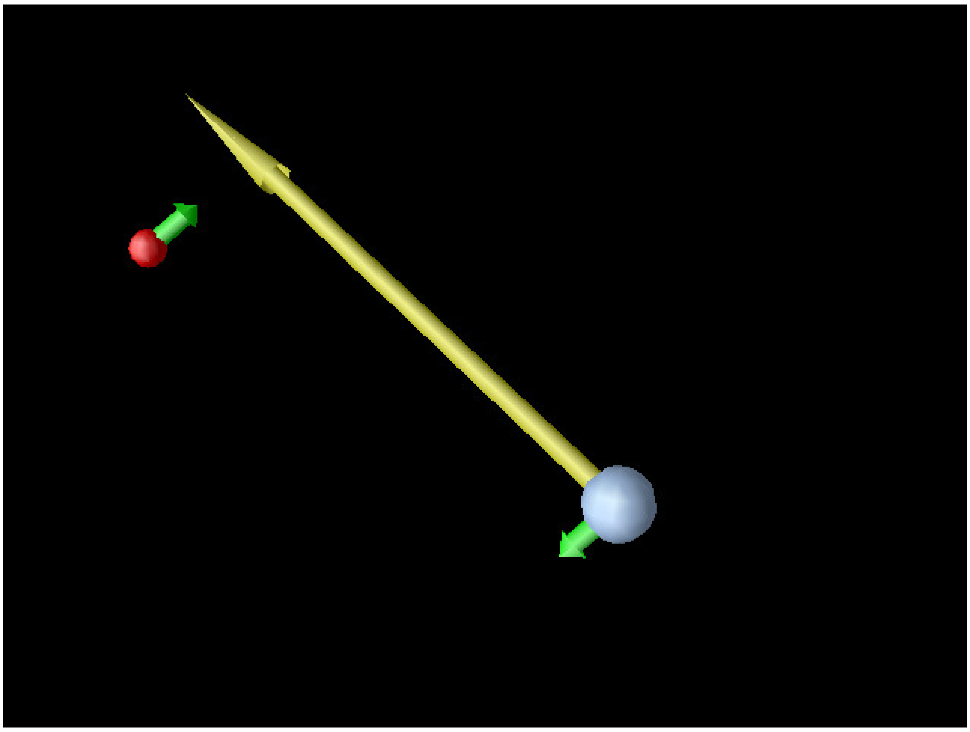

Figure 2: Illustration of the optimization strategy: From the S/C attitude control system and an image, we know the direction vector (yellow) from the S/C (gray sphere) towards a landmark. Unless landmark and S/C positions have converged, the landmark position (red sphere) will be offset from the true viewing direction. In order to reconcile optimized postions with known viewing direction, we pull the landmark towards the line of sight and the S/C in the opposite direction (green arrows).

The S/C positions in iteration k are computed from the positions at iteration $k-1$ by subtracting the mean landmark offset vector for the respective image i (with the scaling factor $\gamma^{(k)}$ applied, cf. Eq. 2):

$$\hat{\mathbf{s}}_i^{(k)} = \mathbf{s}_i^{(k-1)} - \frac{\gamma^{(k)}}{m_{\text{val},i}} \sum_{j \in M_i}^{m} \mathbf{d}_{i,j}^{(k-1)}. \tag{7}$$

We have to estimate also the rotation matrices $R_i^{(k)}$. In order to decompose the landmark offset vectors into a rigid rotation of all landmarks and individual shifts of individual landmarks, we apply Singular Value Decomposition (SVD). In the first step, we compute the 3×3 covariance matrix

$$C_i^{(k)}(p,q) = \sum_{j \in M_i} \tilde{\mathbf{r}}_{i,j}^{(k-1)}(p) \left(\tilde{\mathbf{r}}_{i,j}^{(k-1)}(q) + \frac{\gamma^{(k)}}{n} \mathbf{d}_{i,j}^{(k-1)}(q) \right) \tag{8}$$

for $p, q = 1, \ldots, 3$. For the SVD, we employ the linear algebra package LAPACK [6], in particular the routines DGEBRD, DORGBR, and DBDSQR. The result of the SVD is the rotation matrix $D_i^{(k)}$ that represents the rotation change that best reproduces all desired landmark position changes in a least squares sense, i. e., the updated optimum asteroid rotation for image i is

$$R_i^{(k)} = D_i^{(k)} R_i^{(k-1)}. \tag{9}$$

There is an ambiguity in these rotations. An arbitrary rotation added to all $R_i^{(k)}$ could be compensated by the reverse rotation of the landmarks. In order to avoid a respective drift during the iterations, we fix the rotation for image $i = 1$ and keep for all iterations $R_1^{(k)} = R_1^{(1)}$.

In order to obtain the required residual landmark shifts, we take the shift implied by the rotation change $D_i^{(k)}$, subtract it from the (scaled) intended landmark shift, and revert the asteroid rotation applied for image i. This is for each landmark averaged over all images (in which the landmark is valid):

$$\tilde{\mathbf{d}}_j^{(k)} = \sum_{i \in N_j} R_i^{\mathrm{T}(k-1)} \left(\frac{\gamma^{(k)}}{n_{\mathrm{val},j}} \mathbf{d}_{i,j}^{(k-1)} - \left(D_i^{(k)} \tilde{\mathbf{r}}_{i,j}^{(k-1)} - \tilde{\mathbf{r}}_{i,j}^{(k-1)} \right) \right), \tag{10}$$

with

$$N_j = \{ i \,|\, i \in \{1, \ldots, n\} \wedge j \in M_i \} \tag{11}$$

and $n_{\mathrm{val},j} = |N_j|$. We apply these landmark shifts to obtain the updated positions

$$\hat{\mathbf{r}}_j^{(k)} = \mathbf{r}_j^{(k-1)} + \tilde{\mathbf{d}}_j^{(k)}. \tag{12}$$

We consider landmark and S/C positions relative to the center of all landmarks. This is what we know, we do not know the true center of mass of Didymos. This introduces an additional wobble with Didymos' rotation period and a radius given by the distance between the center of the landmarks and the center of mass, but it turns out that it can easily be separated from the actual wobble which has a different period, Dimorphos' orbital period.
We compute the center of all landmarks for the current iteration

$$\mathbf{r}_{\mathbf{c}}^{(k)} = \frac{1}{m} \sum_{j=1}^{m} \hat{\mathbf{r}}_j^{(k)} \tag{13}$$

and use it to re-normalize the landmark and S/C positions to be relative to the landmark center.
From only the images, we do not have information on the absolute scale. The asteroid could be small and close or large and far. We assume that we have at least one absolute distance measurement, e. g., from the laser altimeter, for at least one landmark in at least one image, and this provides a scale factor $a^{(k)}$.
Because of the updates from the previous iteration $k-1$ to the current iteration k, the landmark center and the scale of everything might have slightly changed, therefore we re-normalize all landmark and S/C positions:

$$\mathbf{r}_j^{(k)} = a^{(k)} (\hat{\mathbf{r}}_j^{(k)} - \mathbf{r}_{\mathbf{c}}^{(k)}), \tag{14}$$

$$\mathbf{s}_i^{(k)} = a^{(k)} (\hat{\mathbf{s}}_i^{(k)} - \mathbf{r}_{\mathbf{c}}^{(k)}). \tag{15}$$

This concludes iteration k. We have computed $R_i^{(k)}$, $\mathbf{s}_i^{(k)}$, and $\mathbf{r}_j^{(k)}$, which enter into the next iteration.

4 DISCUSSION

In our approach, we only use the landmark positions in images and the knowledge from the S/C attitude control. We do not use any additional information, e. g., from S/C tracking. We also do not apply any modeling of asteroid rotation or S/C propagation. Each image is treated separately.
This rather barebone approach could demonstrate that it is possible to retrieve the wobble of Didymos with at least the accuracy required by the AIM mission [1]. With the additional data and modeling implied by a full flight dynamics analysis like it will be conducted for operation of the Hera mission, the accuracy will be even better. However, the approach presented herein could be interesting also for other applications. One could be the reconstruction of position and orientation of an irregularly behaving object caused by outgassing or rotational excitation after an impact.

REFERENCES

[1] B. Grieger and M. Küppers, "Determining the mass of Didymos' secondary by visual imaging," in *EGU General Assembly 2016*, ser. Geophysical Research Abstracts, vol. 18, 2016.

[2] P. Michel, M. Küppers, A. C. Bagatin, *et al.*, "The ESA Hera Mission: Detailed Characterization of the DART Impact Outcome and of the Binary Asteroid (65803) Didymos," *Planetary Science Journal*, vol. 3, no. 7, 160, p. 160, Jul. 2022. DOI: `10.3847/PSJ/ac6f52`.

[3] A. F. Cheng, H. F. Agrusa, B. W. Barbee, *et al.*, "Momentum transfer from the DART mission kinetic impact on asteroid Dimorphos," *Nature*, vol. 616, no. 7957, pp. 457–460, Apr. 2023. DOI: `10.1038/s41586-023-05878-z`. arXiv: `2303.03464 [astro-ph.EP]`.

[4] R. T. Daly, C. M. Ernst, O. S. Barnouin, *et al.*, "Successful kinetic impact into an asteroid for planetary defence," *Nature*, vol. 616, no. 7957, pp. 443–447, Apr. 2023. DOI: `10.1038/s41586-023-05810-5`.

[5] C. A. Thomas, S. P. Naidu, P. Scheirich, *et al.*, "Orbital period change of Dimorphos due to the DART kinetic impact," *Nature*, vol. 616, no. 7957, pp. 448–451, Apr. 2023. DOI: `10.1038/s41586-023-05805-2`. arXiv: `2303.02077 [astro-ph.EP]`.

[6] *LAPACK: Linear Algebra PACKage*. [Online]. Available: `https://www.netlib.org/lapack/`.

[7] *SPICE: An observation geometry system for space science missions*, NASA's Navigation and Ancillary Information Facility (NAIF). [Online]. Available: `https://naif.jpl.nasa.gov/naif/`.