Near Earth Object Deflection Formulae

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Weapon Simulation & Computing Program

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Abstract

If a Near Earth Object (NEO) is discovered on an impact trajectory with Earth, a deflection mission would need to be planned. The planetary defense community practices planning such missions so they can be prepared. While a kinetic impactor is the first choice for a deflection mission, either a large asteroid or a short warning time may require using a stand-off nuclear device to deflect the NEO. A stand-off nuclear device emits x-rays that heat the surface layer of the NEO and causing it to vaporize and blowoff which imparts momentum to the NEO. Several years ago, a simple formula to estimate the δv imparted to a NEO by a stand-off nuclear explosion was developed and informally distributed for use in planning NEO deflection missions. This work will give the derivation of that formula. In addition, the formula will be extended to properly handle the low fluence limit. This covers the case where the whole irradiated surface of the NEO is not melted. The original formula also did not account for how the angle of incidence lowers the deposition scale length. This keeps the energy closer to the surface and reduces the depth to which material is melted. Finally, another formula based on the impulse developed by the energy deposition profile is compared to the previous formula.

These formulae cannot predict the blowoff momentum from first principles. This is because the blowoff momentum depends on the shape, composition, and structure of the NEO, which will almost certainly be poorly known. In addition, because the x-rays deposit their energy on a length scale measured in microns, the surface will be heated enough to reradiate some of the deposited energy. Therefore, the coefficients in these formulae are fit to the results of radiation-hydrodynamic calculations done for a grid of stand-off distances and yields. These calculations are required to account for the reradiation of energy by the surface and for thermal waves that propagate into the material before the hydrodynamic blowoff gets fully underway.

Mission planners need a “fast” estimate of Δv

- For a scenario with a short warning time or a large asteroid mass kinetic impactor missions may not generate sufficient $\Delta \boldsymbol{v}$ to avoid a collision with Earth.
- A nuclear deflection/disruption mission may be able to generate the required $\Delta \boldsymbol{v}$.
- To accurately estimate the $\Delta \boldsymbol{v}$ requires a full simulation using a radiation-hydrodynamics code on a supercomputer. These simulations take a long time to run.
- A “fast” formula to estimate $\Delta \boldsymbol{v}$ is needed by mission planners and scenario generators to consider many mission options.

Outline

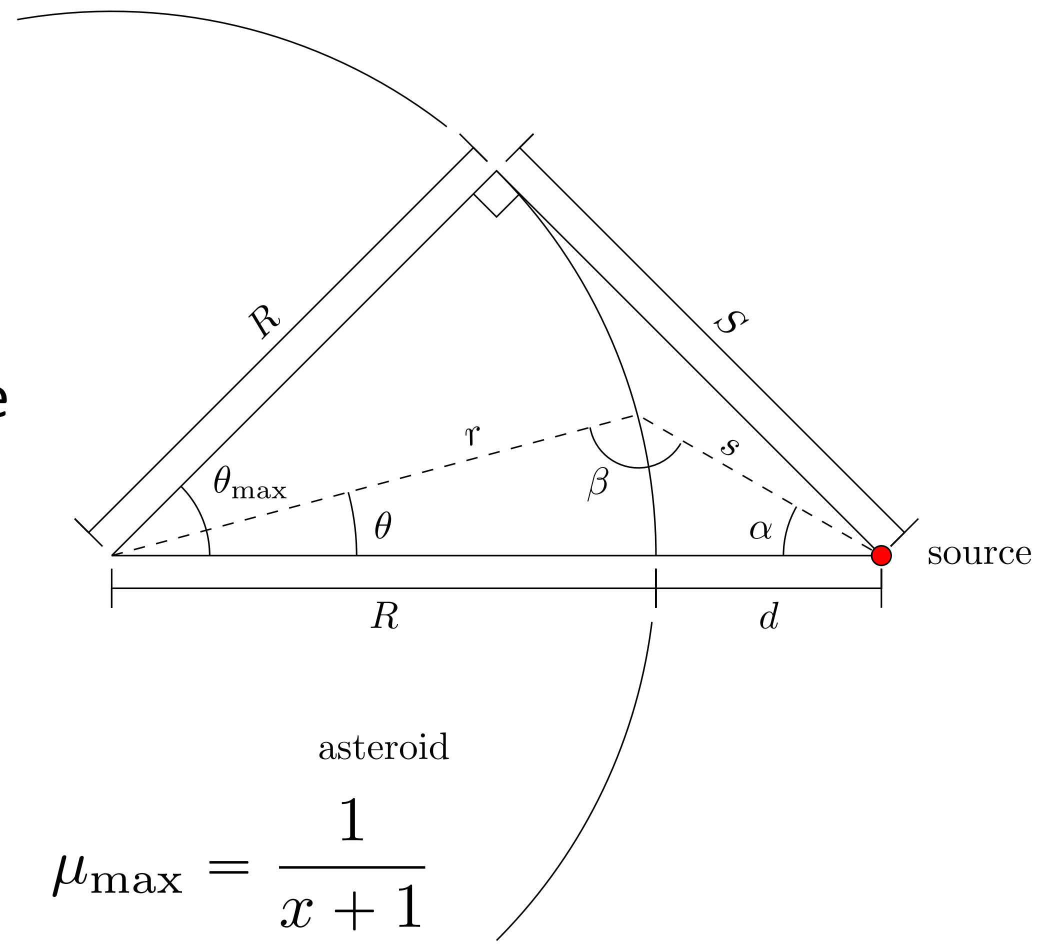
- Derive the original formula
- Extend it to cover the low fluence case
- Modify it to properly account for the effect of the angle of incidence on the deposition depth
- Present an impulse-based semi-analytic formula.
- Compare these formulae
- Future work

The original formula

- Several years ago Joe Wasem brought together our work and provided such a formula
- It is based on the idea of calculating the mass that will be melted as a minimum estimate of the mass that will be ejected.
- This mass and the energy deposited gives a one-point estimate of the momentum of the ejected material.

The geometry

- The geometry and the variables used are in this figure.
- s is the distance from the source to the surface of the asteroid.
- Let $x = \frac{d}{R}$, then $\frac{s^2}{R^2} = 1 + (x+1)^2 - 2(x+1)\mu$
- $\pi - \beta$ is the angle of incidence.



$$\mu_{max} = \frac{1}{x+1}$$

Deposition Profile

- We use an exponential deposition profile for the energy per volume.

- If the fluence at the surface is $\frac{Y}{4\pi s^2}$ then

$$\varepsilon(z) = \frac{Y}{4\pi\lambda_d s^2} e^{-z/\lambda_d} \;\Rightarrow\; z_{\mathrm{melt}} = \lambda_d \ln[Y/(4\pi\rho\lambda_d\epsilon_{\mathrm{melt}} s^2)]$$

- It is convenient to let $b = 4\pi\rho\lambda_d\epsilon_{\mathrm{melt}}$ which represents the fluence required to have the surface melt.

- We introduce two dimensionless variables that the deflection formulae depend on, $x = d/R$ and $y = Y/(bd^2)$. The latter must be > 1 for there to be any melt and ejected mass.

The original formula

- We are using the one-point approximation $\delta v = \sqrt{2 M_{\rm ej} E_{\rm dep}} / M_{\rm ast}$

$$M_{\rm ej} = 2\pi R^2 \int_{\mu_{\rm max}}^{1} \rho \lambda_d \ln\left(\frac{Y}{bs^2}\right) d\mu$$

$$= \pi \rho \lambda_d R^2 \frac{x^2}{x+1} \left\{ \frac{2}{x}\left[1 + \ln y\right] - \left(1 + \frac{2}{x}\right) \ln\left(1 + \frac{2}{x}\right) \right\}$$

$$E_{\rm dep} = \frac{Y}{2}(1 - \cos\alpha_{\rm max}) = \frac{Y}{2}\left[1 - \sqrt{1 - (x+1)^{-2}}\right]$$

$$\delta v = \frac{3}{4R^2}\sqrt{\frac{Y\lambda_d}{\pi\rho}} \sqrt{\frac{x^2}{x+1}\left\{ \frac{2}{x}\left[1 + \ln y\right] - \left(1 + \frac{2}{x}\right) \ln\left(1 + \frac{2}{x}\right) \right\} \left[1 - \sqrt{1 - (x+1)^{-2}}\right]}$$

- The leading coefficient is made into a fitting parameter

$a = \frac{3}{4}\sqrt{\frac{\lambda_d}{\pi\rho}}$. All the units remain in $\frac{a\sqrt{Y}}{R^2}$

Extending the formula to low fluence

- For small values of y the formula will be wrong since z_{melt} becomes negative for small values of μ. This is true when $y < \dfrac{x+2}{x}$
- The solution is to use the value of μ where $z_{\mathrm{melt}} = 0$ as the lower limit in the mass integral and for calculating E_{dep}

$$\mu_1 = 1 - \frac{x^2(y-1)}{2(x+1)}$$

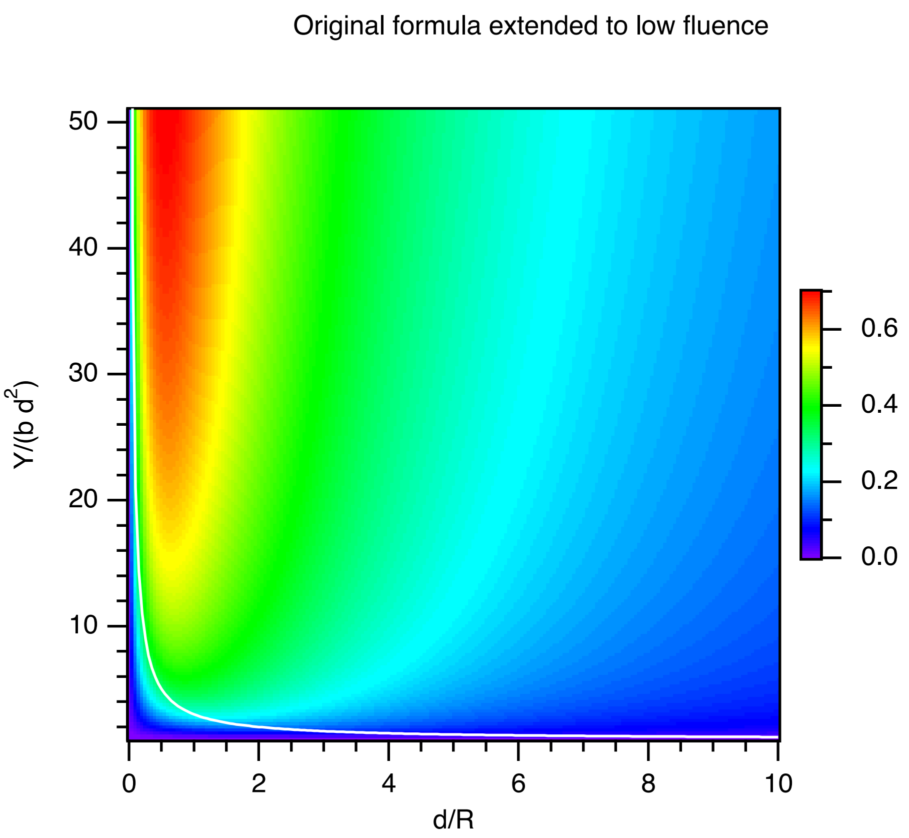
$$M_{\mathrm{ej}} = \pi\rho\lambda_d R^2 \frac{x^2}{x+1}\left(y - 1 - \ln y\right)$$

$$E_{\mathrm{dep}} = \frac{Y}{2}\left(1 - \sqrt{1 - \frac{x^2}{y}\left[\frac{y-1}{2\,(x+1)}\right]^2\left[\frac{4\,(x+1)}{x^2(y-1)} - 1\right]}\right)$$

$$\delta v_{\mathrm{low}} = \frac{a\sqrt{Y}}{R^2}\sqrt{\frac{x^2}{x+1}\left(y-1-\ln y\right)\left(1 - \sqrt{1 - \frac{x^2}{y}\left[\frac{y-1}{2\,(x+1)}\right]^2\left[\frac{4\,(x+1)}{x^2(y-1)} - 1\right]}\right)}$$

Shape of the formula

- The figure gives the dimensionless part of δv (removing the $a\sqrt{Y}/R^2$ term)
- When fit to calculations we get $a = 5370, b = 2.16\times10^{-4}$ assuming Y is in kilotons and R is in meters.
- The white line is the boundary below which the low fluence equation is used.



A better melt depth

- The original formula did not account for the fact that when the x-rays hit the asteroid at an angle of incidence the deposition length normal to the surface has a factor of the angle of incidence in it. We let M'_{ej} be the dimensionless part of the mass.

$$\varepsilon(z) = \frac{Y\cos\beta}{4\pi\lambda_d s^2\cos\beta}e^{z/(\lambda_d\cos\beta)} = \frac{Y}{4\pi\lambda_d s^2}e^{z/(\lambda_d\cos\beta)}$$

$$z_{\text{melt}} = \lambda_d\frac{(x+1)\mu-1}{\left[1+(x+1)^2-2(x+1)\mu\right]^{1/2}}\ln\left[\frac{yx^2}{1+(x+1)^2-2(x+1)\mu}\right]$$

$$\begin{aligned} M_{\text{ej}} &= \pi R^2\rho\lambda_d\frac{2}{9(x+1)}\left\{[x(x+2)]^{3/2}\left[8+3\ln\left(\frac{yx}{x+2}\right)\right]-x^2[2(4x+9)+3(x+3)\ln y]\right\} \\ &= \pi R^2\rho\lambda_d M'_{\text{ej}} \end{aligned}$$

$$\delta v = \frac{a_c\sqrt{Y}}{R^2}\sqrt{M'_{\text{ej}}\left[1-\sqrt{1-(x+1)^{-2}}\right]}$$

The low fluence case

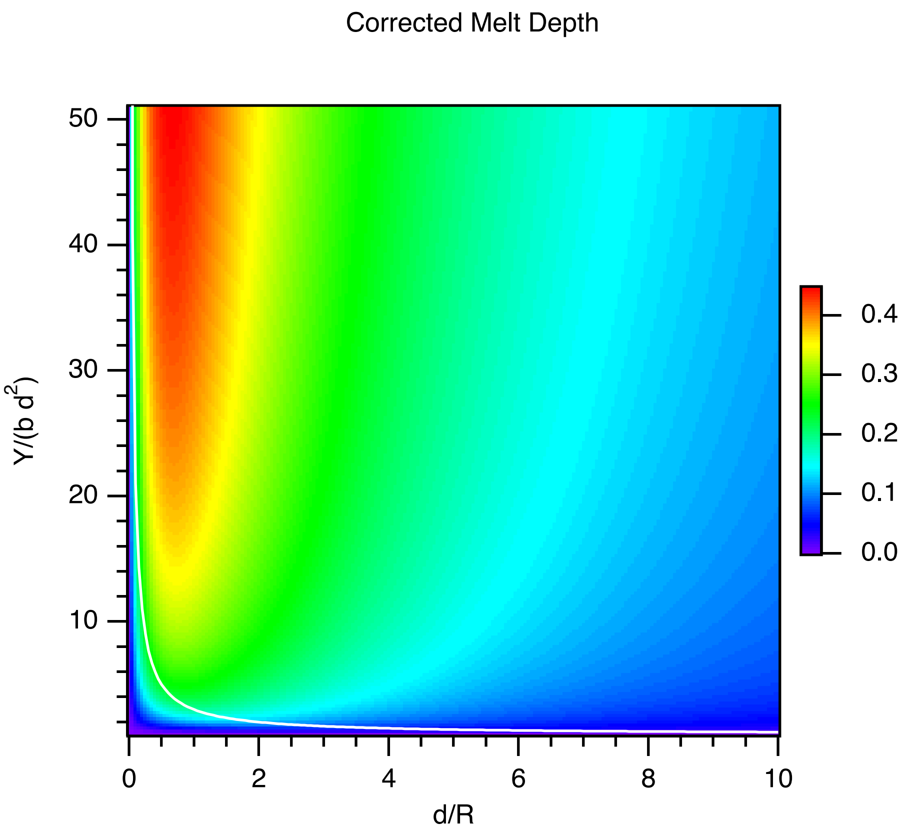
- This modified formula also needs to be extended to low fluence with the same lower limit on μ.

$$
\begin{aligned}
M_{\mathrm{ej}} &= \pi R^2 \rho \lambda_d \frac{2x^2}{9\,(x+1)} \left\{ y^{1/2} \left[9(x+2) - xy \right] - 2\,(4x+9) - 3\,(x+3) \ln y \right\} \\
&= \pi R^2 \rho \lambda_d M'_{\mathrm{ej}}
\end{aligned}
$$

$$
\delta v_{\mathrm{low}} = \frac{a_c \sqrt{Y}}{R^2} \sqrt{ M'_{\mathrm{ej}} \left(1 - \sqrt{ 1 - \frac{x^2}{y} \left[\frac{y-1}{2\,(x+1)} \right]^2 \left[\frac{4\,(x+1)}{x^2 (y-1)} - 1 \right] } \right) }
$$

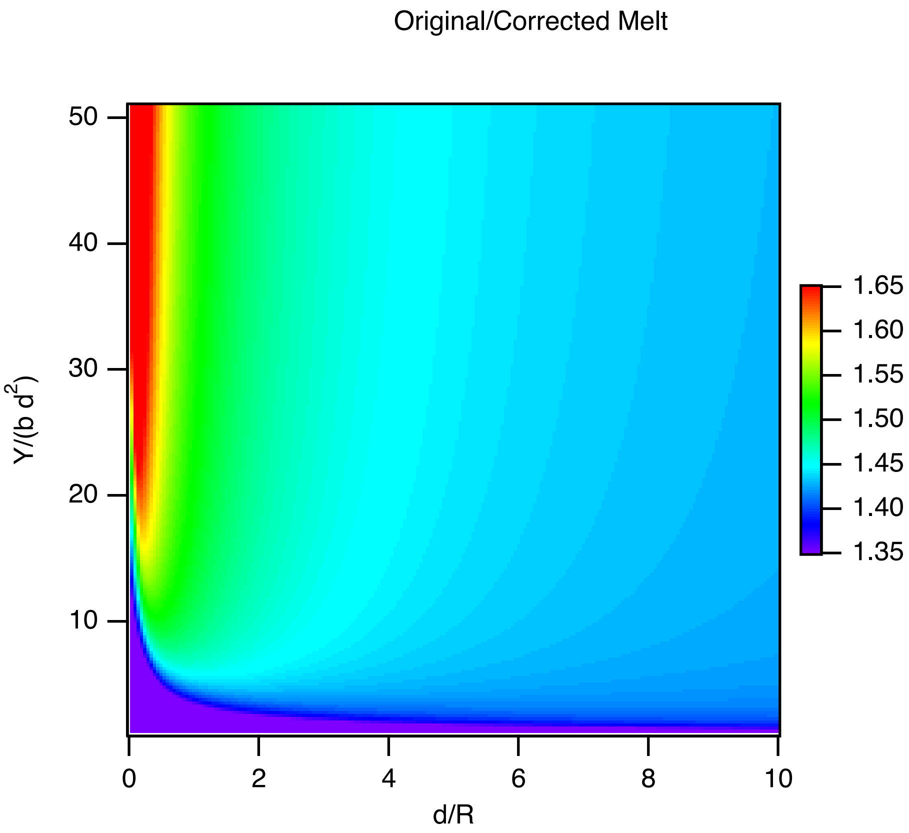
Corrected melt depth

- The corrected melt depth is shallower so less mass is ejected and the momentum and hence the velocity change will be smaller.
- The general shape of the function is very similar.
- When fit to calculations we get $a_c = 10020, b_c = 3.7 \times 10^{-4}$
- The white line is the boundary below which the low fluence equation is used.



Comparison of the two formulae

- The ratio between these two formulae shows where the difference is.
- The rise for $d < R$ is different and the low fluences show differences.
- The bulk of the plot is very similar with only a small gradient of differences.



An impulse-based formula

- To further improve on the formula requires moving away from the one-point nature of it.

- Another model[1] takes the energy deposition profile and assumes that in each slice normal to the surface the energy in excess of the melt energy turns into kinetic energy.

$$I(\mu) = \sqrt{2}\int_0^{z_{\mathrm{melt}}} \rho\sqrt{2[\epsilon(z) - \epsilon_{\mathrm{melt}}]}\, dz$$

$$F_0^* = -\frac{F_0}{\rho\epsilon_{\mathrm{melt}}\lambda_d \cos\beta} = \frac{Y}{b_I s^2}$$

$$I(\mu) = -\frac{a_I\sqrt{Y}}{R}\rho\frac{4\sqrt{2}}{3x\sqrt{y}}\cos\beta\left[\sqrt{F_0^* - 1} - \tan^{-1}\left(\sqrt{F_0^* - 1}\right)\right]$$

[1] B.P. Shafer, M. D. Garcia, R. J. Scammon, C. M. Snell, R. F. Stellingwerf, J. L. Remo, R. A. Managan and C. E. Rosenkilde. The coupling of energy to asteroids and comets, in T. Gehrels, M.S. Matthews, A. Schumann (Eds.) Hazards Due to Comets and Asteroids, vol 24, University of Arizona Press, 1994, pp. 955-1012.

An impulse-based formula

- We integrate this impulse over angles where $F_0^* \geqslant 1$ which ensures there is melt at the surface.

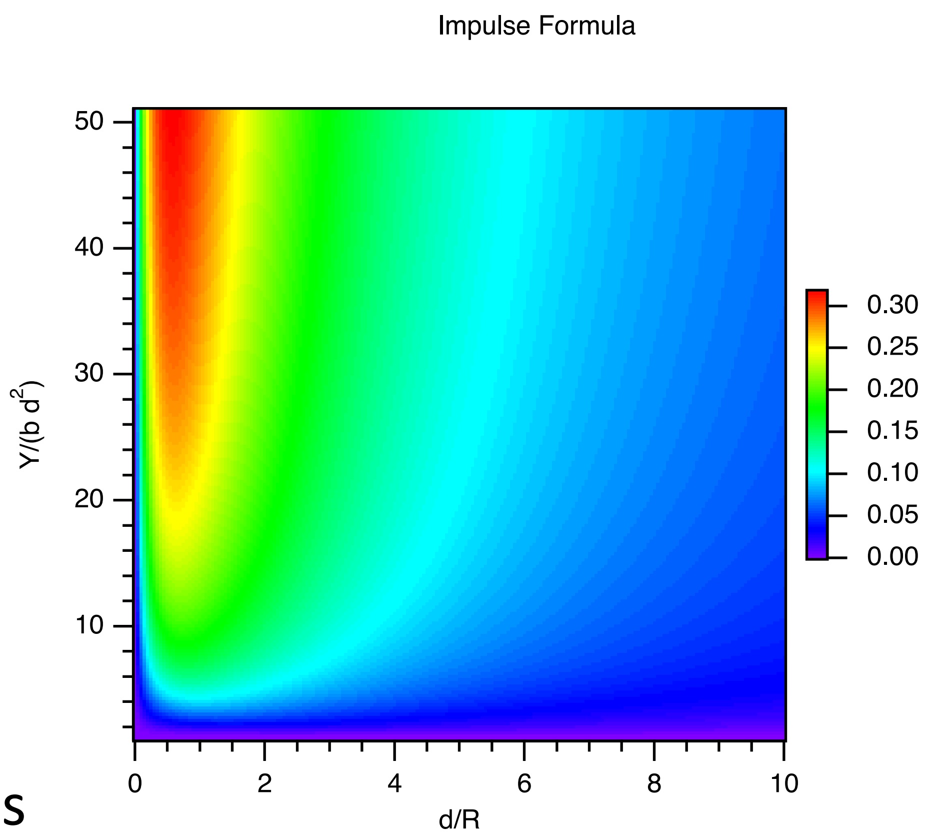
$$\delta v = \frac{I_{tot}}{\frac{4}{3}\pi R^3 \rho} = \frac{3}{2R\rho}\int_{\mu_1}^{1} I(\mu)\,\mu\,d\mu$$

$$= \frac{a_I\sqrt{Y}}{R^2}\frac{2\sqrt{2}}{x\sqrt{y}}\int_{\mu_1}^{1}\frac{[(x+1)\mu-1]\left[\sqrt{F_0^*-1}-\tan^{-1}\left(\sqrt{F_0^*-1}\right)\right]}{[1+(x+1)^2-2(x+1)\mu]^{1/2}}\,\mu\,d\mu$$

- While this integral can not be evaluated analytically it is not hard to do so numerically.

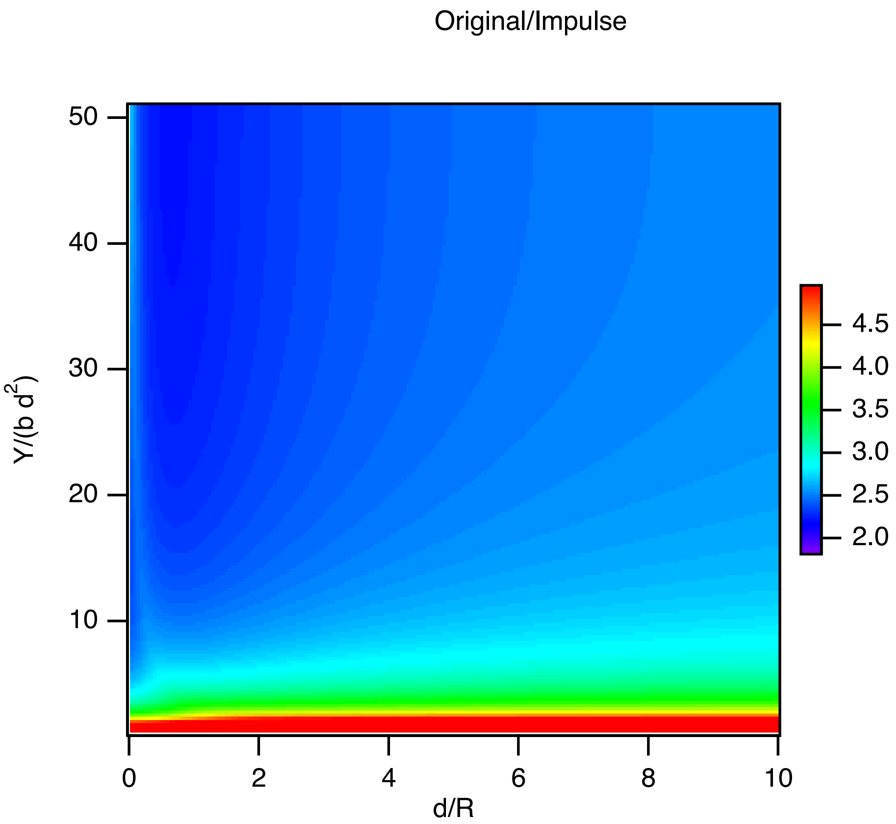
The impulse-based formula

- This formula again gives a similar shape with a lower magnitude.
- The lower magnitude has two causes. First the energy below melt is not included and only the component of the momentum along the axis is integrated.
- When fit to the existing simulations we get poor results for the coefficients. We continue to work on this.



Comparison

- Comparing the impulse formula to the original shows a larger change in magnitude but less change in the shape.
- There is a noticeable reduction for small fluences, presumably because the melt energy is a large fraction of the incident energy in that case.



Future work

- Detailed radiation-hydrodynamic calculations for a broader sample of values for x and y.
- At this time we recommend the original formula with the coefficients $a = 5370, b = 2.16\times10^{-4}$.
- Extend the calculations to materials other than SiO_2.
- Reradiation of the heated asteroid material needs to be accounted for directly.

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