ABSTRACT

This paper proposes a monolithic and a distributed optimization architecture for the structure and robust control co-design of a flexible spacecraft in presence of uncertainties. The novelty consists in applying the multi-body Two-Input Two-Output Port (TITOP) theory to model flexible sub-structures in a minimal Linear Fraction Transformation (LFT) form by including parametric uncertainties. In this way the time-consuming iterations due to the traditional sequential approach adopted in industry (optimization of the structure and then of the controller) and to the validation and verification (V&V) control campaigns are shortcut. The LFT framework allows in fact to include directly in the controller synthesis the set of uncertainties and eventually of the mechanical parameters to be optimized (for monolithic optimization). As consequence, the V&V phase benefits of formal proofs provided by $\mu$-analysis, which considerably limits the amounts of Monte Carlo’s time domain simulations. The proposed approach is applied to a real benchmark, the ENVISION spacecraft, for which a considerable reduction of mass is achieved by coping with the imposed control performance and a large set of uncertainties.

1 INTRODUCTION

The widespread approach for system design and optimization problems adopted in the Space industry generally follows a sequential logic by neglecting the interconnection among different disciplines. However, since the optimization objectives in the different fields are often conflicting, this methodology can fail to find global optimal solutions. By restricting the analysis to just structure and control fields, the common hierarchy is to preliminary define the structure by optimizing the physical design parameters and then leave the floor to the control optimization. This process can be iterated several times before a converging solution is found and control performance is met. Especially for large flexible structures, the minimization of the structural mass corresponds in fact to an increase in spacecraft flexibility, by bringing natural modes to lower frequencies where the interaction with the Attitude and Orbit Control System (AOCS) can be critical, especially in the presence of system uncertainties. Modern Multidisciplinary Design and Optimization (MDO) techniques nowadays represent a tool to enhance the optimization task by integrating in a unique process all the objectives and constraints coming from each field. Two kinds of architectures can be distinguished in the MDO framework:
monolithic and distributed. In a monolithic approach, a single optimization problem is solved, while in a distributed architecture the same problem is partitioned into multiple sub-problems containing smaller subsets of the variables and constraints. The development in the last decade of structured $H_{\infty}$ control synthesis opened the possibility of robust optimal co-design of structured controllers and tunable physical parameters. In fact, LFT formalism allows some sizing mechanical parameters to be considered as decision variables exactly at the same level than the gains of the structured controller to perform robust mechanical/control co-optimization. In addition, thanks to these techniques, particular properties can be imposed on the controller, such as internal stability or performance respecting a frequency template, in the face of all the parametric uncertainties of the plant (also taken into account with LFT). This point is particularly important for aerospace applications where requirements are generally challenging and structural uncertainty, coming for example from an imperfect manufacturing or assembly, cannot be neglected. It has to be said that these techniques do not guarantee a global optimal solution of problem, so a good first guess can enhance the quality of the result. Alazard et al. \cite{1} demonstrated how this multi-model methodology implemented in $H_{\infty}$ framework can be enlarged to include integrated design of system sizing physical parameters and the structured controller. There exist as well in literature a large class of problems where coupling between structure and control is considered unidirectional. This means that the objective function of the structural sub-problem depends only on the structural design parameters while the control criterion depends on both structural and control design parameters. A partition of the structure and controller design variables is desirable for practical implementation when the impact of the controller variables on the structural objective is relatively small. A strategy in this case is to solve the system-level problem as a nested optimization one, as in the BIOMASS test case \cite{2}. For the present study both monolithic and distributed architectures are investigated on a real benchmark, the ENVISION spacecraft preliminary design. In particular, the problem formulation in the multi-body Two-Input Two-Output Ports (TITOP) \cite{3} modeling approach allows the author to easily define an MDO problem by including all possible system uncertainties from the very beginning of the spacecraft design. In this way not only a structure/control co-design is possible, but system performance is robustly guaranteed. Where an analytical model of the structure is sufficient to describe the various spacecraft sub-components, a dependency from the design parameters can be captured in a minimal LFT model (built in SDTlib \cite{4}). In this approach the control/structure co-design problem is solved in a unique iteration by using the non-smooth techniques available in the Robust Control community. When the complexity of a structure cannot be handled with a simple analytical model (i.e. finer Finite Element Model (FEM) are necessary to ensure representativeness), a distributed architecture will be preferred. A nested optimization process is in fact necessary when a FEM software such as NASTRAN has to be interfaced with the control synthesis/analysis tools available within MATLAB/SIMULINK. In this case, the strategy is to iteratively optimize an inner $H_{\infty}$-control problem, which depends on both control and structural design variables, and the structural design themselves are optimized by an outer global optimization routine. The aim of this paper is finally to contribute to the evolution of industrial practice in control/structure co-design, by proposing a unified and generic approach based on a well-posed modeling problem that integrates both design parameters and parametric uncertainties in a unique representation. The advantage offered by this framework is dual: to shortcut the unnecessary iterations among different fields of expertise and to speed up the validation and verification process by directly producing a robust preliminary design.
2 SPACECRAFT MODELING

One of the main goals of this paper is to propose an innovative methodology to overcome the common industrial practice in the optimization process of structure and control design. The workflow generally followed in industry is the one proposed in Fig. 1a: the optimization of structures and control laws is separately done by two different entities which rarely interact by exchanging information in the same nomenclature conventions. Structural models provided by the structure department to the AOCS team are generally very preliminary and an important uncertainty margin has to be taken into account in the control synthesis in order to cover possible evolution of the structural design till its final version. Rarely the optimized design of the structure architecture is updated to reinitialize the control synthesis and analysis process for cost/time reasons. This brings then to conservative control designs that could result in sub-optimal solutions. This problem becomes more and more important if very fine pointing accuracy is demanded. The alternative process proposed in this study is shown in Fig. 1b. In this case complex parameterized FEM models are considered in the control problem from the beginning of the design. A parallel optimization (monolithic or distributed) of structures and control laws is performed. Moreover the use of the LFT framework allows the introduction of uncertainties in the model used for control synthesis such that the final controller already robustly cope with them. This methodology thus also optimizes the number of control syntheses generally done in common industrial practice, which are often based on the nominal mechanical model and some worst-cases (WC) dictated by experience. The V&V process could finally benefit of the proposed approach since powerful tool for uncertain closed-loop analysis are nowadays available. These methods, as $\mu$-analysis, can analytically prove the design robustness and detect very rare worst-case that could escape to Monte Carlo sample-based simulation approach.

2.1 ENVISION benchmark

Let consider the ENVISION spacecraft depicted in Fig. 2. It is constituted by a rigid central body $B$, two solar arrays $S_1$ and $S_2$, a Subsurface Radar System (SRS) composed of two flexible beams $Q_1$ and $Q_2$, and a Synthetic Aperture Radar (SAR) $V$. In Fig. 2 it can be noticed that the central body reference frame is centered in its center of mass (CoM) $B$, while all appendages have their body frame defined in correspondence of their attachment nodes. Both Simscape and SDTlib tools are used in this study to model the ENVISION benchmark in the 6 d.o.f. case. The particularity is to have a structured model in which each sub-structure with its own sizing parameters can be isolated as well as local mechanisms or joints linking the sub-structures. The SDTlib is used to build the entire dynamical model of the spacecraft as illustrated in Fig. 3a in a SIMULINK environment for control synthesis and worst-case linear analysis. Note that the modeling in this framework allows the user to have the entire system already in a minimal LFT form. Two ways to model each sub-structure are available in the toolbox:

- Complex flexible bodies (as the solar panels with sandwich-structured composite) can be modelled by directly integrating the results of a FEM modal analysis
- Simple flexible bodies (i.e. the two booms of the SRS) or local mechanisms (i.e. Solar Array Drive Mechanism, reaction wheels and gearboxes)

The most common tool for FEM analysis within the Space industry is NASTRAN. In terms of interfacing the NASTRAN physical FEM model into the advanced AOCS modeling framework, the current baseline is reduction of the FEM to modal matrix form using NASTRAN core SUPERELEMENT capabilities and importing these matrices into MATLAB through a purpose written MATLAB
script. NASTRAN is able to make accurate estimations of the elastic and inertial property definitions of an elastic system i.e. a structure using FEM. It is able to re-characterize large finite element models consisting of thousands of elements into a set of relatively small matrices containing mass, stiffness and mode shape information (Craig-Brampton modeling) to accurately reproduce the mass and stiffness matrices of the linear elastic system under consideration.

For these reasons an automatic interface between SDTlib and NASTRAN has been developed with the possibility to add to the NASTRAN model (passed to SDTlib through a BDF and an F06 file) uncertainties on flexible mode frequencies, participation factors, damping factors and modal shapes. In this way, by starting from a nominal NASTRAN model a large family of uncertain plant can be easily derived and used for robust control synthesis and analysis.

A way to include in Simscape environment accurate NASTRAN FEM models is to use the Reduced Order Flexible Solid (ROFS) block. In [5] the authors present how to automatically generate in SDTlib the inputs for this Simscape block directly from NASTRAN input/output files. The Simscape

Figure 1: Structure/control optimization process: (a) common industrial practice, (b) methodology proposed in this study

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model of Envision benchmark is shown in Fig. 3b. The comparison in frequency domain between SDTlib and Simscape models of ENVISION is provided in Fig. 4. In particular the nominal $3 \times 3$ transfer function between the 3 external torques acting at the central body CoM and the consequent 3 angular acceleration of this point is depicted.

The trade-off on the modeling tools considered in this study is finally presented in Table 1.

3 ROBUST CONTROL PROBLEM

The objective of this section is to present the general control architecture of the ENVISION Attitude Control System (ACS). We consider the robust design of a 3-axis structured attitude control law to meet:

- **(Req1)** the absolute pointing requirement, defined by the $3 \times 1$ vector of Absolute Performance Error ($\text{APE} = [0.08, 0.2, 0.08] \cdot 10^{-3}$ rad), in spite of low frequency orbital disturbances dominated by the gravity gradient torque (characterized by the $3 \times 1$ upper bound on the magnitude $T_{\text{ext}} = [1.9, 1.9, 1.9] \cdot 10^{-3}$ Nm),

- **(Req2)** the relative pointing requirement, defined by the $3 \times 1$ vector of Relative Performance Error ($\text{RPE} = [0.5, 0.5, 0.5] \cdot 10^{-3}$ rad) to be kept for a time window $\Delta t_{\text{RPE}} = 15$ s,

- **(Req3)** the maximum command requirement, defined by the $3 \times 1$ vector of maximum control torque ($\bar{u} = [0.215, 0.215, 0.215] \cdot$ Nm),

- **(Req4)** stability margins characterized by an upper bound $\gamma$ on the $\mathcal{H}_\infty$-norm of the input sensitivity function,
Figure 3: ENVISION dynamical model: (a) SDTlib, (b) Simscape

Figure 4: Comparison of SDT and Simscape models of the transfer function \( [W_{\text{ext/B,B}}]_{RB} (4 : 6) \rightarrow [\ddot{x}_B]_{RB} (4 : 6) \)

while minimizing (req5) the variance of the APE and RPE in response to the star sensor and gyro noises characterized by their Power Spectral Density (PSD), respectively \( PSD^{SST} = (3.5 \cdot 10^{-5})^2 I_3 \text{rad}^2/s \) and \( PSD^{GYRO} = (1.4 \cdot 10^{-6})^2 I_3 \text{rad}^2/s \) (assumed to be equal for the 3 components). The value \( \gamma = 1.5 \) ensures on each of the 3 axes:
<table>
<thead>
<tr>
<th><strong>Modeling Needs</strong></th>
<th><strong>Adapted modeling Solution</strong></th>
<th><strong>Adapted tool</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Detailed description of the sub-structure or particular properties of the materials (such as the anisotropy of sandwich solar panels) are considered as design parameters of the co-design process</td>
<td>FEM</td>
<td>Interface Simulink/NASTRAN available in SDTlib</td>
</tr>
<tr>
<td>Taking into account simple mechanical properties, like the length or the cross-section properties of a homogenous beam, or it is possible to easily replace non-isotropic material properties with equivalent isotropic analytical models of beams and plates</td>
<td>Analytical</td>
<td>Set of analytical models available in SDTlib: beams, plates, mechanisms (joints, reaction wheels, solar array drive mechanisms, etc.), simplified dynamics (sloshing effects)</td>
</tr>
<tr>
<td>Modeling of parametric uncertainties</td>
<td>Analytical</td>
<td>In all STDlib features, parametric uncertainties can be taken into account (included models obtained by FEM subsystems) in order to build minimal LFT models</td>
</tr>
<tr>
<td>Simulation of non-linear rigid dynamics</td>
<td>Non-causal approaches</td>
<td>Simscape allows multi-physical modeling. Time simulation is appreciable when rigid dynamics is considered</td>
</tr>
<tr>
<td>Simulation of linear time invariant (LTI) or Linear parameter-varying (LPV) flexible dynamics</td>
<td>FEM/Analytical</td>
<td>Linearization of SDTlib model in form of LTI/LPV systems</td>
</tr>
</tbody>
</table>

Table 1: Trade-off of modeling methods

- a disk margin $> 1/\gamma = 0.667$,
- a gain margin $> \frac{\gamma}{\gamma-1} = 3 (9.542 \text{ dB})$,
- a phase margin $> 2 \arcsin \frac{1}{\gamma} = 38.9 \text{ deg}$.

The requirements **Req1** to **Req4** must be met for any values of the uncertain mechanical parameters regrouped in the block $\Delta^{SC}$.

The closed-loop architecture considered for the ENVISION’s attitude control system is shown in Fig. [5](#). In this figure the uncertain plant is represented by the nominal block $[M^{SC}]_{R_B}(s)$, that represents exactly the open-loop system in Fig. [3a](#). The block $\Pi^{SC}$ includes all the isolated optimization mechanical parameters modeled as uncertainties in SDTlib. This block, as better explained in the following sections, is used only in the case of the monolithic optimization, since for the distributed...
optimization one, the mechanical parameters will be fixed at each nested control synthesis. The avionics is composed by 3 Reaction Wheels (RW(s)) system, a 3-axes Gyro (GYRO(s)), a 3-axis Star Tracker (SST(s)) and a loop delay (DELAY(s)). The input weighting filters W_{ext}, W_{GYRO} and W_{SST} respectively normalize the input external torque perturbation, the Gyro and the Star Tracker Power Spectral Densities (PSD). The output weighting filters finally translate (Req1) to (Req5).

The MATLAB routine systune available in the Robust Control Toolbox [6] allows to easily solve the control optimization problem by finding the best controller K_{ACS}(s) with a Proportional-Derivative imposed structure that meets all requirements and copes with the whole set of model uncertainties.

4 STRUCTURE/CONTROL CO-OPTIMIZATION

As already anticipated in the previous sections, robust control synthesis can directly be used for a monolithic optimization implementation. The flowchart in Fig. 6 outlines the principal steps in this approach. The key point is to be able to build an LFT model which includes all possible admissible ENVISION’s dynamics. This corresponds to obtaining the uncertain block \( \Pi^{SC} \) in Fig. 5 to be directly optimized in the control synthesis. A way to obtain this uncertain model is to interpolate several models (analytical or FEM) of the ENVISION’s flexible appendages computed for different combinations of the mechanical optimization parameters in their admissible ranges. The APRICOT library [7] offers many routines to achieve this objective.

If a distributed optimization is used instead, the corresponding iterative process is depicted in Fig. 7. The chosen global optimization routine in this study is the Particle Swarm Optimization (PSO), implemented in the MATLAB Global Optimization Toolbox. The difference with the monolithic optimization is that at each iteration a new set of the mechanical optimization parameters is chosen based on the previous iterations and a new robust control problem is solved. The algorithm provides
the optimal mechanical and control solution given by the minimization of the cost function:

\[
\hat{J} = \min (\alpha J_m + \beta J_c)
\]  

(1)

where \(J_m\) is the performance index associated to the structural optimization problem (the mass of the overall spacecraft in this study), \(J_c\) is the performance index associated to the control optimization problem (directly provided by systune) and \(\alpha\) and \(\beta\) are weighting factors.

5 RESULTS AND DISCUSSION

The general performance of the two optimization approaches applied to the ENVISION benchmark are summarized in Table 2. Even if the achieved optimized mass is similar with the two approaches, the distributed optimization allow a large number of optimization parameters to be considered (with a consequent lower computational time), in comparison with the monolithic optimization. This number is in fact limited by the complexity of the interpolated LFT, that can contain a huge number of uncertain repeated parameters, which can make the robust control optimization infeasible. In this study four parameters out of twelve have been chosen based on their impact on the overall spacecraft mass.
This choice constrains the achievable control performance, that is driven by other parameters. As shown in Table 3, the distributed optimization brings to a better pointing and stability performance.

![Figure 7: Distributed optimization flowchart](image)

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Monolithic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimization total time (h)</td>
<td>12.33</td>
</tr>
<tr>
<td>Optimal spacecraft mass (kg)</td>
<td>1253.86</td>
</tr>
<tr>
<td>Optimized spacecraft mass (kg)</td>
<td>55.94</td>
</tr>
<tr>
<td>% Optimized spacecraft mass</td>
<td>4.27%</td>
</tr>
<tr>
<td>Optimal max control performance</td>
<td>0.7208</td>
</tr>
</tbody>
</table>

Table 2: General performance of distributed and monolithic optimization

The final step of the end-to-end co-design process is the validation of the control performance with formal proof provided by a μ-analysis (available in the wcgain routine of the MATLAB Robust Con-
Table 3: Optimized control performance with distributed and monolithic optimization

<table>
<thead>
<tr>
<th>Cont. Req.</th>
<th>Distributed</th>
<th>Monolithic</th>
</tr>
</thead>
<tbody>
<tr>
<td>APE</td>
<td>0.7208 ($\bar{\mu}_\Delta = 0.7353$)</td>
<td>1.0011 ($\bar{\mu}_\Delta = 1.00051$)</td>
</tr>
<tr>
<td>RPE</td>
<td>0.0583</td>
<td>0.1016</td>
</tr>
<tr>
<td>Command</td>
<td>0.0122</td>
<td>0.0157</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>0.7208 ($\bar{\mu}_\Delta = 0.7530$)</td>
<td>1.0011 ($\bar{\mu}_\Delta = 1.01$)</td>
</tr>
<tr>
<td>Noise Variance</td>
<td>0.0469</td>
<td>0.0521</td>
</tr>
</tbody>
</table>

As shown in Fig. 8 and Fig. 9 $\mu$-analysis confirms the worst-case performance directly detected in the control synthesis by robustly proving the performance and stability of the system. Note that the analysis is driven for different configurations of the solar panels angular position ($\theta_{SA}$).

A final trade-off on the optimization algorithms used in this work is provided in Table 4.

6 CONCLUSION

In this paper two different optimization architectures are proposed for the structure and control co-optimization of a large flexible space structure. In particular it has been shown how the applied modeling framework, the TITOP approach, that takes into account all possible parametric uncertainties and optimization variables in a unique minimal LFT form, can be efficiently exploited to this goal. The direct application of the proposed method to the ENVISION benchmark finally allows the authors to obtain a consequent reduction of the overall spacecraft mass by meeting the needed control performance.
Figure 9: Worst-case analysis of control performance for monolithic optimization

<table>
<thead>
<tr>
<th>Architecture</th>
<th>PRO</th>
<th>CONS</th>
</tr>
</thead>
</table>
| Monolithic   | • Structure and Control design parameters at the same optimization level  
|              | • “Fast” control re-design (only one control synthesis needed)  
|              | • Long preliminary generation of a family of models + APRICOT  
|              | • Limited amount of design parameters (synthesis/analysis algorithms sensitivity to number of uncertain repetitions)  
| Distributed  | • Large number of structure design parameters possible  
|              | • Higher possibility to not fall into local optimal solutions  
|              | • Structure and Control design parameters not at the same optimization level  
|              | • Not fast control re-design (n nested control design needed)  

Table 4: Trade-off of optimization architectures

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