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OPTIMAL TRAJECTORY DESIGN OF ASTEROID CAPTURE DURING CLOSE ENCOUNTER WITH EARTH

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ABSTRACT

Nowadays, more attention has been paid to Near Earth Objects (NEOs), and there are more than 2000 asteroids in the current list of NEOs. In order to better defend against these NEOs' impacts, more in-depth understanding of NEOs is needed. In this paper, a fictious mission of capturing a NEO into the Earth-Luna system for further asteroid exploration is considered. In the trajectory design process, the trajectory of the selected NEO in the heliocentric segment will be analysed using the high-fidelity dynamical model in General Mission Analysis Tool (GMAT). The first maneuver will be imposed to push the asteroid into the Earth's sphere of influence (SOI) by establishing and solving a typical Lambert Problem. After that, the NEO should be analyzed in the Earth-Luna system and to ensure the asteroid's being captured in a relatively stable trajectory, its Jacobian constant will be constrained within an appropriate range after the second maneuver. Numerical simulation is performed to check the stability of the trajectory after capture. The final minimum velocity increment derived from the analysis of the selected example is 462.68 m/s.

1.Introduction

Near Earth Objects (NEOs) have been attracting considerable attention and more and more NEOs have been detected in the recent years. Among these NEOs, the potentially hazardous objects (PHOs) have a greater chance of impacting the earth and causing danger. More attention has been paid to the research on PHOs, not only because of their potential threat of an impact on earth but also because of the abundance of resources in these asteroids.

Many deep space exploration missions have been carried out to observe and obtain the physical properties of these PHOs[1][2]. However, deep space exploration missions will take a long time and involve huge efforts, it would be much easier to observe and study these PHOs if they could be captured directly near Earth. Many research works related to the capture mission have been done in recent years[3][4]. In this paper, we will introduce a capture scheme which divides the capture mission into two parts: heliocentric segments and geocentric segments, and a more accurate three-body model is used to calculate the velocity increment required for capture.

First, we can obtain the Modified Keplerian of the PHO and its original trajectory in the heliocentric coordinate can be analyzed by using the General Mission Analysis Tool (GMAT). The rough intersection time of the PHO and the Earth can be obtained in GMAT and the velocity and position vector of this interval can be output.

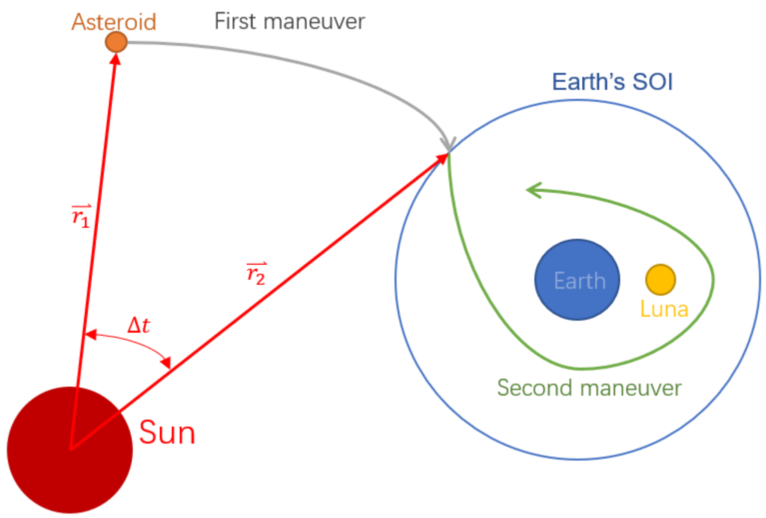
Despite our efforts to search, we still have not found a suitable PHO that can directly access the Earth's SOI . As a result, the first maneuver needs to be carried out to

push the asteroid into the Earth's SOI. The initial and terminal position vectors are known, and by changing the time of flight, the velocity increment of the first maneuver can be obtained by solving a typical Lambert problem.
The second maneuver occurs after the asteroid enters the Earth's SOI, and by calculating and limiting its Jacobi constant, it can be captured inside the allowable regions of motion in the Earth-Luna system. The captured trajectory will be obtained by numerical integration, and the second velocity increment can also be obtained by a simple calculation of the PHO's Jacobi constant. Adding the two velocity increments together, we can obtain the total velocity increment required for the capture scheme.

2.Dynamical model

2.1 Lambert Problem

Solar gravity plays a major role in influencing the trajectory of the asteroid when it does not enter the Earth's SOI. As a result, the first maneuver needs to be added to the asteroid so that it can fly from a point in the heliocentric system towards Earth and enter the Earth's SOI directly, as is shown in $Fig. 1$.



$Fig. 1$ Schematic of the capture.

When the position vectors of the asteroid ($\vec{r_1}$) and of the Earth ($\vec{r_2}$) from the Sun are known, by setting the time of flight (Δt), the deltaV required for the first maneuver can be obtained, using the solution of the Lambert Problem.

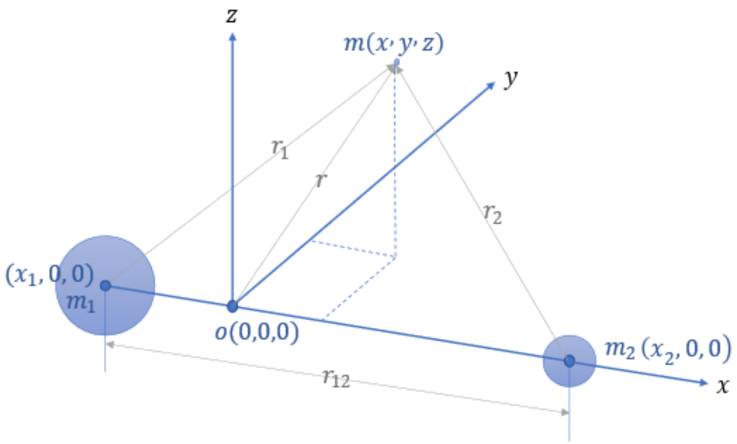
$$\vec{v_1} = \frac{1}{g}(\vec{r_2} - f\vec{r_1}) \tag{1a}$$

$$\vec{v_2} = \frac{1}{g}(\dot{g}\vec{r_2} - \vec{r_1}) \tag{1b}$$

The f and g in (1) can be represented by the global variable χ[5][6], and the derivation gives f and $\dot{g}$. Then the velocity vector $\vec{v_1}$ and $\vec{v_2}$ corresponding to $\vec{r_1}$ and $\vec{r_2}$ can be obtained.

2.2 Restricted Three-body Model and Jacobian Constant

As the asteroid gradually approaches the Earth from the heliocentric system, the sun's gravitational influence on the asteroid is getting weaker. When the asteroid is less than 0.3 AU away from the Earth, the analyze system is transformed into the Earth-Luna-Asteroid three-body system.
To better illustrate the asteroid's trajectory in the restricted three-body system, a rotating coordinate system needs to be introduced here. In this coordinate, the origin is at the center of mass of the Luna-Earth system, the x-axis points from the origin to the center of mass of the Luna (m_2), the y-axis lies in the orbital plane of the Luna-Earth system, and the z −axis is perpendicular to the $x-y$ plane.



$Fig.$ 2 Earth-Luna rotating coordinate.

Assuming that the Earth-Luna system rotates at a uniform angular velocity Ω, the mass of earth is m_1, Luna's is m_2, G is the gravitational constant.

$$\Omega = \sqrt{\frac{\mu}{r_{12}^3}} \tag{2}$$

$$M = m_1 + m_2 \tag{3}$$

$$\mu_1 = Gm_1 \tag{4a}$$

$$\mu_2 = Gm_2 \tag{4b}$$

$$\mu = \mu_1 + \mu_2 \tag{4c}$$

Define two dimensionless values π_1, π_2.

$$\pi_1 = \frac{m_1}{m_1 + m_2} \tag{5a}$$

$$\pi_2 = \frac{m_2}{m_1 + m_2} \tag{5b}$$

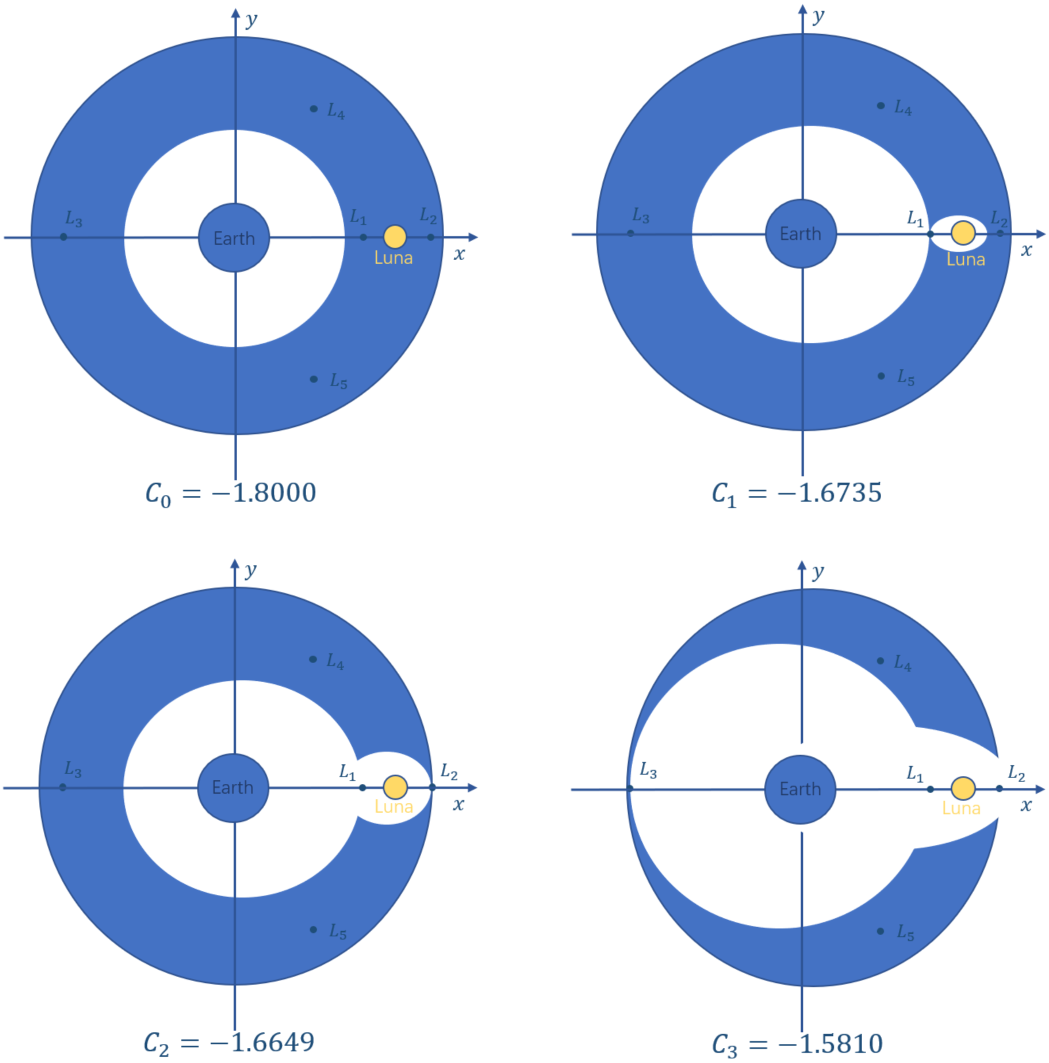
The three scalar equations for the restricted three-body system can be obtained as follows [7].

$$\ddot{x} - 2\Omega\dot{y} - \Omega^2 x = -\frac{\mu_1}{r_1^3}(x + \pi_2 r_{12}) - \frac{\mu_2}{r_2^3}(x - \pi_1 r_{12}) \tag{6a}$$

$$\ddot{y} + 2\Omega\dot{x} - \Omega^2 y = -\frac{\mu_1}{r_1^3} y - \frac{\mu_2}{r_2^3} y \tag{6b}$$

$$\ddot{z} = -\frac{\mu_1}{r_1^3} z - \frac{\mu_2}{r_2^3} z \tag{6c}$$

In the restricted three-body model, the well-known Jacobian constant C can be used to describe the total energy possessed by the third body with respect to the rotating coordinate [8].



$Fig.3$ Allowable regions for different C in Earth-Luna system.

As is shown in $Fig.3$, to have an asteroid captured by the Earth-Luna system, one only needs to limit its Jacobian constant to between C_0 and C_2.

3.Capture Example

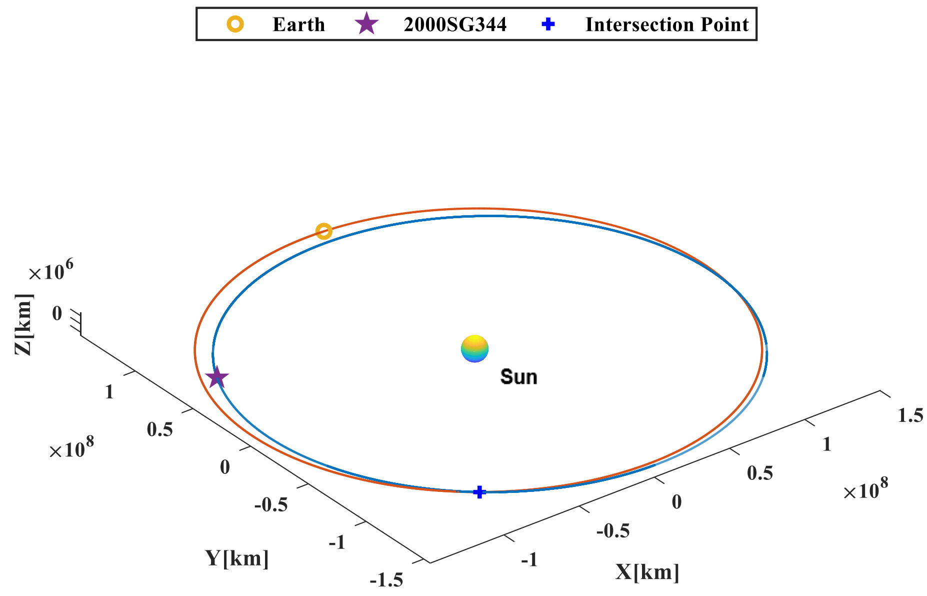
Taking 2000SG344 as a capture example, its Keplerian elements in Heliocentric coordinates are shown in **Table 1**.

Table 1 Keplerian elements of 2000SG344(Epoch: MJD 29200.5)*

SMA (km)	ECC	INC (deg)	RA (deg)	AOP (deg)	TA (deg)
146219950.0954	0.066932	0.112	191.892	275.385	236.392

* https://newton.spacedys.com/neodys/

Construct the Sun-Earth-Asteroid three-body dynamical model and substitute the Keplerian elements into the integrator. The trajectory of the heliocentric segment can be obtained as $Fig.4$.



$Fig.4$ The trajectories of 2000SG344 and Earth in the heliocentric system.

The point where the asteroid intersects the Earth can be obtained on $Fig.4$ and the approximate time is calculated as MJD 2461932.4.
In order to obtain more precisely vectors of the asteroid's velocity and position at the intersection time, the Keplerian elements of the asteroids need to be substituted into the General Mission Analysis Tool (GMAT). The rough rendezvous time previously obtained in MATLAB is selected, and 200 days before it is used as the integration interval to obtain the 6400 sets of velocity and position vector values of 2000SG344 when located near the Earth (32 sets of data for a day).
The coordinate transformation from the heliocentric system to the Earth-Luna rotating coordinate is used to output the required values to calculate the Minimum Orbital Intersection Distance (MOID) with the Earth, as is shown in **Table 2**.

Table 2 Position vector near the closest point to Earth

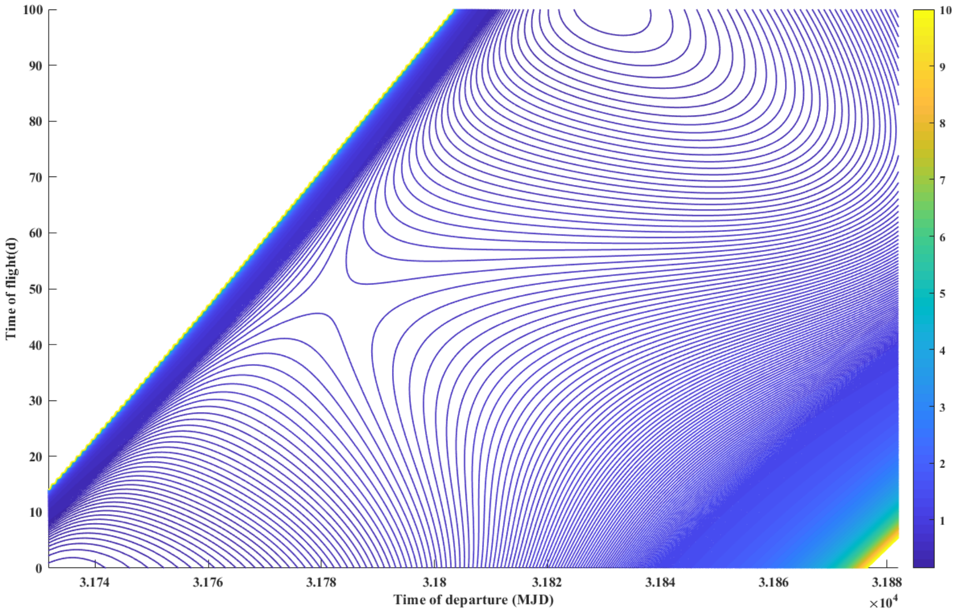
Epoch (MJD)	r_x(km)	r_y(km)	r_z(km)
2461932.3437504	554701.514736	-3085168.240031	-1047791.40405
2461932.3750004	551728.240694	-3085641.432852	-1047945.04692
2461932.4062504	548754.234674	-3086115.590064	-1048099.20578
2461932.4375004	545779.497320	-3086590.713418	-1048253.88154
2461932.4687504	542804.029283	-3087066.804672	-1048409.07511

As the MOID is calculated by the values in **Table 2**, 2000SG344 has the closest point to Earth which is still 3305110.072km (0.022AU) away. So, the first maneuver needs to be imposed on 2000SG344 to bring it within the radius of the Earth's SOI. The position vector of the asteroid in the heliocentric system is known as $\overrightarrow{r_1}$, the position vector of the Earth in the heliocentric system can be obtained from the ephemeris as $\overrightarrow{r_2}$, and the time-of-flight interval is chosen to be 50 days before and after the date of the asteroid's original flight to perigee without maneuvering, which is a changing Δt.

According to (1), the $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ corresponding to the changing Δt can be obtained. The initial velocity of the asteroid $\overrightarrow{v_0}$ is available in the output of GMAT.

$$\Delta v_1 = |\overrightarrow{v_1} - \overrightarrow{v_0}| \tag{7}$$

The speed increment of the first maneuver can be obtained by (7). With these elements, a Pork Chop Chart with departure time and flight time as bivariate can be drawn .



$Fig.5$ Velocity increment required for the first maneuver.

In $Fig.5$, the minimum velocity increment can be found, which is 0.14315km/s. After the first maneuver, 2000SG344 has flown from perigee within the Earth's SOI, thus breaking away from the influence of the Sun's gravity on the trajectory and beginning the second phase: the restricted three-body motion in the Earth-Luna-Asteroid system.

When 2000SG344 enters the Earth's SOI after the first maneuver, its absolute velocity equals $\overrightarrow{v_2}$. According to the residual velocity equation and the absolute velocity equation:

$$\overrightarrow{v_\infty} = \overrightarrow{v_2} - \overrightarrow{v_e} \tag{8}$$

$$\overrightarrow{v_a} = \overrightarrow{v_r} + \vec{\Omega} \times \overrightarrow{r_r} \tag{9}$$

$\overrightarrow{v_a}$ is the absolute velocity of 2000SG344, which equals $\overrightarrow{v_\infty}$, $\overrightarrow{v_e}$ is the absolute velocity of the Earth's motion, $\vec{\Omega}$ is the rotational angular velocity of the Earth-Luna rotating coordinates, $\overrightarrow{r_r}$ is the relative position vector of 2000SG344 to the Earth. According to (8) and (9), the velocity of the asteroid relative to the Earth-Luna system as it enters the Earth's SOI can be obtained as follows:

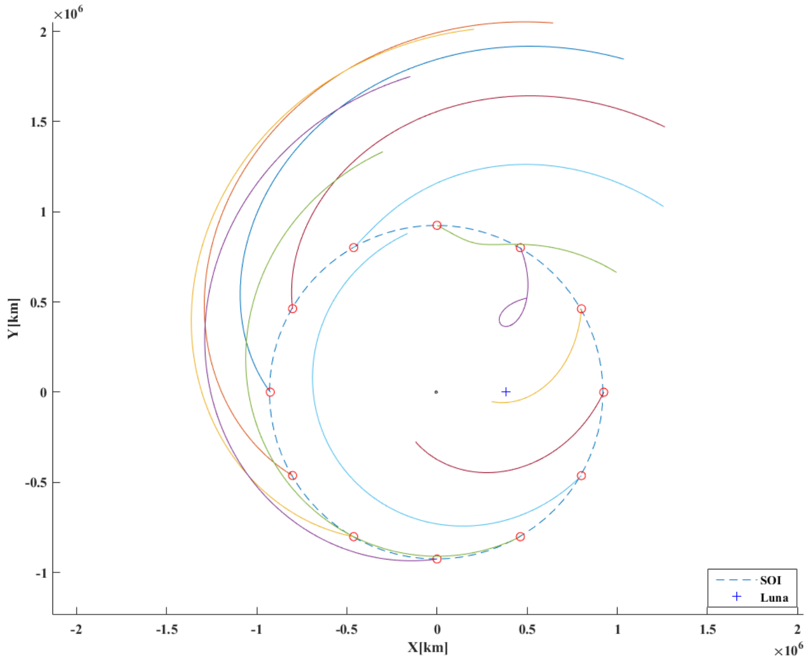
$$\overrightarrow{v_r} = \overrightarrow{v_2} - \overrightarrow{v_e} - \vec{\Omega} \times \overrightarrow{r_r} \tag{10}$$

Substituting the $\overrightarrow{v_r}$ and $\overrightarrow{r_r}$ into the integrator and using the $x-y$ plane of the Earth-Luna rotating coordinate as the orbital plane, which requires eliminating the velocity component in the z –axis (0.1054787898km/s). The trajectory of 2000SG344 can be obtained as $Fig.6$, plotted at 30-degree intervals.

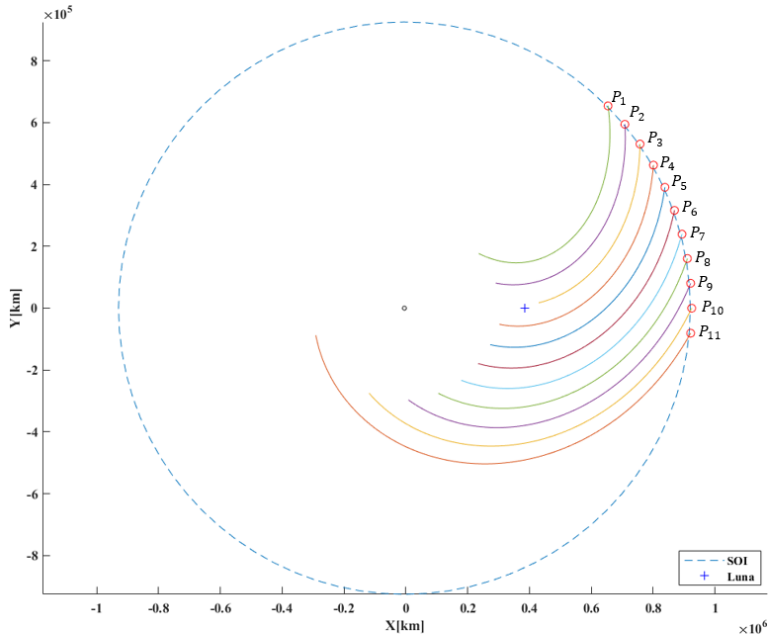
When 2000SG344 enters the Earth-Luna rotating coordinate, its Jacobian constant can be used to analysis its allowable regions as is shown in $Fig. 3$.

$$v^2 = \Omega^2(x^2 + y^2) + \frac{2\mu_1}{r_1} + \frac{2\mu_2}{r_2} + 2C \tag{11}$$

Related parameters refer to $Fig. 2$ and $(1) - (4)$.
According to $Fig. 6(a)$, the entry positions which cause the flight away from the Earth's SOI should be deleted, the rest divided by a more precise 5-degree intervals, shows in $Fig. 6(b)$.



(a)



(b)

$Fig. 6$ The trajectory of 2000SG344 after the first maneuver.

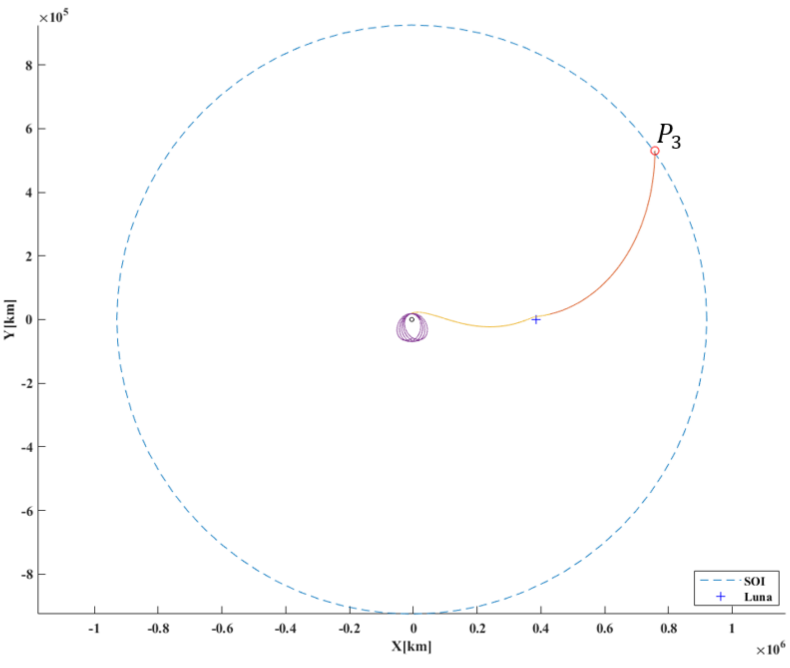
As is marked from P_1 to P_{11}, the Jacobian constant of these eleven entry positions can be calculated according to (11). Then the increment of velocity in the second maneuver can be obtained as **Table 3**.

Table 3 The increment of velocity for 11 positions in $Fig.6$.

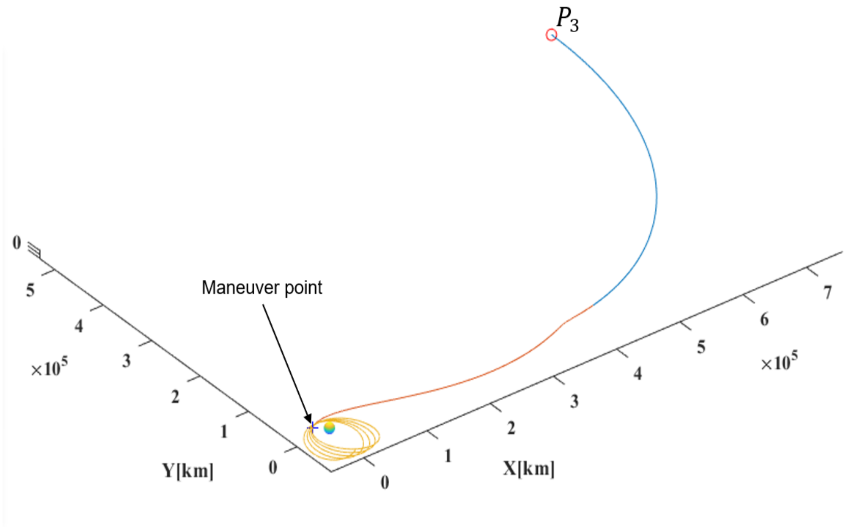
Position	The deltaV in the 2nd maneuver (km/s)	The total deltaV (km/s)
P_1	1.017342454	1.265972067
P_2	0.783093714	1.031723327
P_3	0.214050010	0.462679623
P_4	0.609679960	0.858309572
P_5	0.335962005	0.584591618
P_6	0.527457512	0.776087125
P_7	0.422720011	0.671349623
P_8	0.923680201	1.172309813
P_9	1.451125765	1.699755377
P_{10}	1.999106961	2.247736573
P_{11}	2.687000395	2.935630008

According to the data shown in **Table 3**, the minimum value of the velocity increment obtained at point P3, which is 0.462679623 km/s.

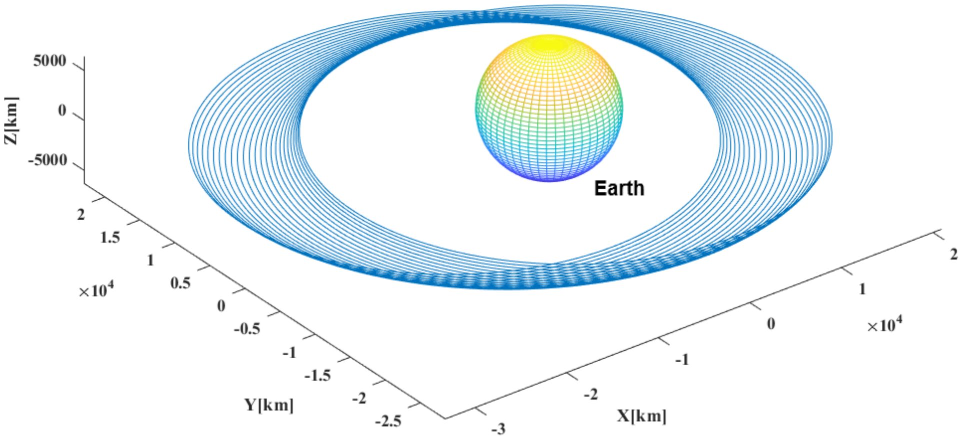
Next step, the velocity and position vectors of 2000SG344 can be substituted into the restricted three-body dynamics model and its captured trajectory after the last maneuver can be drawn, as is shown in $Fig.7$.



(a)



(b)



(c)

$Fig.7$ The trajectory of 2000SG344 after capture mission.

At the end of the last maneuver, 2000SG344 is firmly captured in an approximately elliptical trajectory about 12,000 km above the Earth's surface, as is shown in $Fig.7(c)$, where 2000SG344 will orbit long enough to let people observe and study its physical properties, with a total velocity increment of around 462 m/s.

4.Conclusion

This paper attempted to investigate the capture mission of PHOs and try to take advantage of the PHO’s approaching to the Earth for capture. Although the asteroid chosen in this preliminary work does not fly directly into the Earth’s SOI, it should be possible to capture it near the Earth if a PHO is found in the future that can fly naturally into the Earth's SOI with a relatively low velocity using the method studied in this paper.

5.Reference

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