ORBIT PROPAGATION AROUND SMALL BODIES USING SPHERICAL HARMONIC COEFFICIENTS OBTAINED FROM POLYHEDRON SHAPE MODELS

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Introduction

Missions to asteroids have been the trend in space exploration for the last years. They provide information about the formation and evolution of the Solar System, contribute to direct planetary defense tasks, and could be potentially exploited for resource mining. Be their purpose as it may, the factor that all these mission types have in common is the challenging dynamical environment they have to deal with. The gravitational environment of a certain asteroid is most of the times not accurately known until very late mission phases when the spacecraft has already orbited the body for some time.

Polyhedron shape models help to estimate the gravitational potential with a density distribution assumption (usually constant value) and some optical measurements of the body. These measurements, unlike the ones needed for harmonic coefficient estimation, can be taken from well before arriving at the asteroid's sphere of influence, which allows to obtain a better approximation of the gravitational dynamics much sooner. The disadvantage they pose is that obtaining acceleration values from these models implies a heavy computational burden on the on-board processing unit, which is very often too time-consuming for the mission profile.

The purpose of this research is to develop a tool to obtain spherical harmonic coefficients for different polyhedron shape models. Such coefficients are then used to assess the accuracy of this spherical harmonics model w.r.t. the polyhedron one.

From Polyhedra to Spherical Harmonics

The main reference for this work is [1], where the authors developed an algorithm to compute the spherical harmonics coefficients of a given constant density polyhedron. We remark that such a hypothesis is not unrealistic for a wide set of asteroids.

The key idea is to use recurrence relations for the integrands that appear when one computes the spherical harmonics coefficients for a given body. Such integrands usually involve Legendre functions and polynomials. The recurrence relations are presented in both their normalized and non-normalized form.

Finally the polyhedron is partitioned into a collection of simplices and the integration of the integrands is performed. These simplices are tetrahedra whose bases are the triangular faces of the polyhedron and the vertices are at the centre of the reference system used to define the coordinates of the points that form the shape model. A change of variable is used to ease the integration of the aforementioned simplices.

Implementation

In terms of implementation, the main issues identified by [1] are to represent homogeneous polynomials in three variables and to operate with them without using symbolic manipulators. This is achieved by representing trinomials of degree n as arrays of length (n+1)(n+2)/2 whose elements are the coefficients of the trinomials ordered in such a way that each coefficients correspond to the right trinomial.

AstroHarm (AstroSim Harmonics) is a Python suite and module that takes the theory developed on [1] to a software materialisation. The capabilities that this module offers include: wavefront (.obj) files management, geometric assessment of shape models (reference radius, volume, centre of mass…), triplet management and operations, C and S matrixes recursive computation, normalisation routines for coefficients, and file I/O.

Validation

In order to validate the coefficients obtained by AstroHarm, semi-analytical methods were used to obtain spherical harmonic coefficients from accurately known geometric bodies as a cube, a tetrahedron, and a double pyramid.

Using Wolfram Mathematica® software and symbolic formulation, the semi-analytically-computed coefficients were compared to the ones obtained from AstroHarm, and the results were satisfactory.

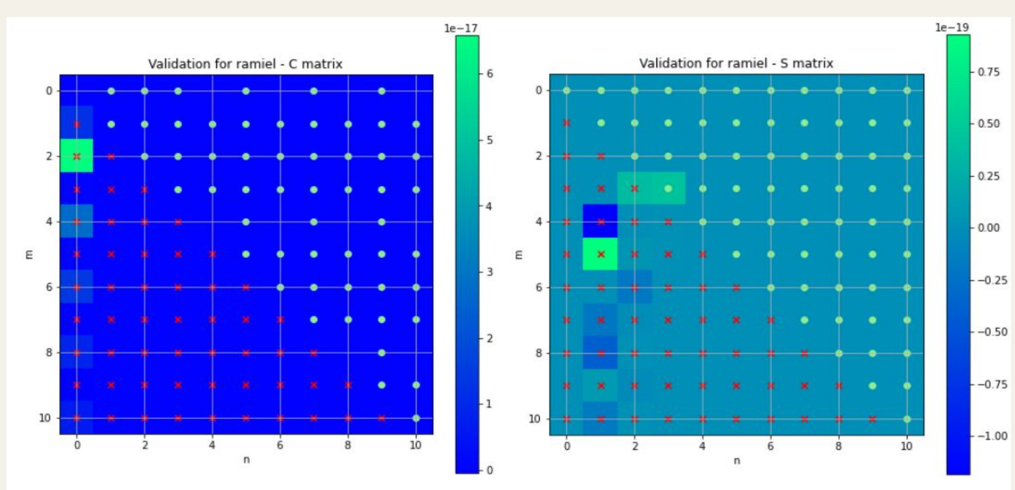

Figure 1: Validation heatmaps for C and S matrixes. Red crosses point out cases were m>n, which should be disregarded due to its lack of conceptual meaning. Green dots represent values that analytically equal to zero. Colorbars talk about the magnitude of the errors, which goes from 1e-17 to 1e-19.

Performance vs Accuracy

Once these coefficients are obtained, a spherical harmonic model has to be implemented. Following the method developed by [2], the gravitational acceleration given by these coefficients can be computed and used as internal dynamics for a given propagator.

To evaluate the accuracy of this spherical harmonics model, a ground truth trajectory is required. This ground truth is obtained from a polyhedron dynamics suite, developed by [3] and based on [4]. This choice is supported by the fact that this method is exact up to the surface of the given shape model, assuming constant density distribution.

The two main drivers of this research are computational performance and results accuracy. Once the computation of the coefficients is validated, the next step is to take the experimentation to a real asteroid for which a shape model is given.

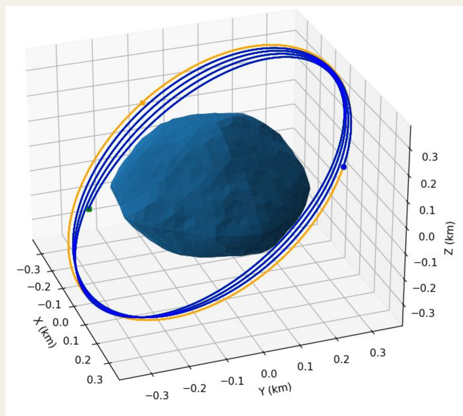

Figure 2: Shape model of asteroid Bennu used for simulation. Orange trajectory: point-mass model; Blue trajectory: polyhedron model; Green trajectories: Spherical harmonics with different n.

Starting with Bennu, shown in the image above, the algorithm is run for different maximum orders for the coefficients going from 2 to 10, obtaining the following results in terms of trajectory difference w.r.t. the polyhedral model, which is taken as ground truth.

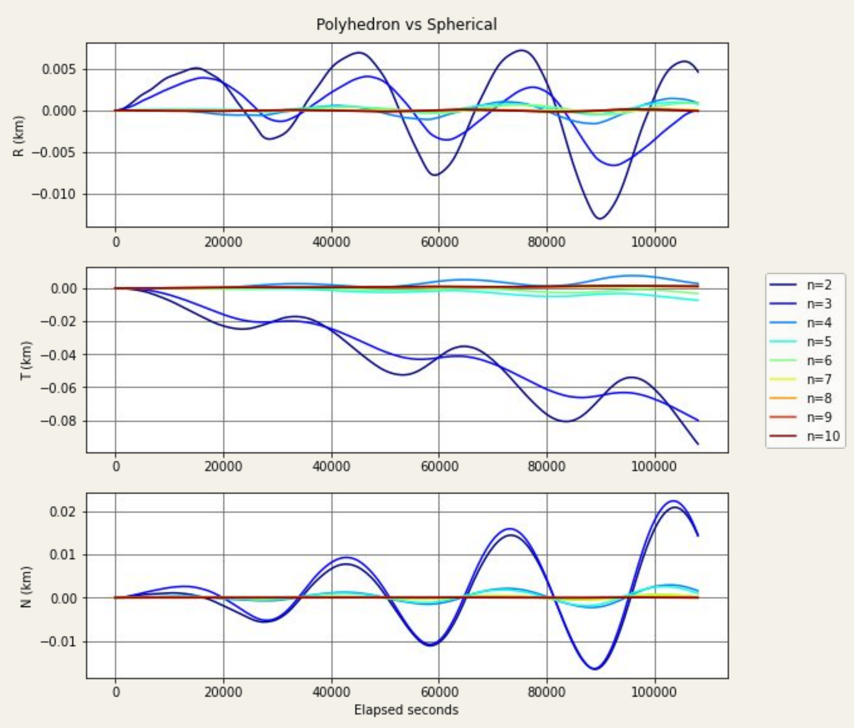

Figure 3: Radial, tangential, and normal component errors for different spherical harmonics models w.r.t. polyhedral model for Bennu. *sma*=500 m (1.73 Bennu radii), ecc=0.1, inc=45º.

It can be clearly seen that, as the order of the coefficients increases, the error is reduced, arriving at values close to no error for n>7 in this case. In terms of computational effort, Figure 4 shows that the computational time is considerably reduced.

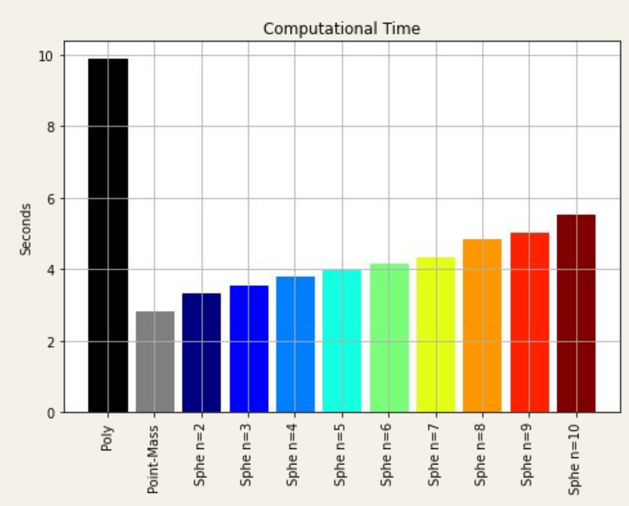

Figure 4: Computational effort for the different values of n after the 30h propagation corresponding to the trajectories shown in Figure 3.

References

[1] R.A. Werner "Spherical harmonic coefficients for the potential of a constant-density polyhedron", Computers Geosciences, vol. 23, no. 10, pp. 1071–1077, 1997.

[2] O. Montenbruck & E. Gill "Satellite Orbits", Springer-Verlag Berlin Heidelberg,2000

[3] D. González, "https://github.com/DaniGlez/polygrav", 2020.

[4] R. A. Werner, "The gravitational potential of a homogeneous polyhedron or Don't Cut Corners", Celestial Mechanics and Dynamical Astronomy,vol. 59, no. 3, pp. 253–278, 1994.

Acknowledgements

The authors would like to thank Cătălin Galeș for his suggestions. The authors would also like to acknowledge the funding received from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 813644.

Limitations: shape and distance

The limitations of this algorithm were also explored within this project. In particular, two specific parameters were investigated: body shape and orbital distance.

The former tried to gain some insight into how the oblateness of a body could make it more difficult to obtain a set of spherical harmonics coefficients that could get as accurate as the polyhedron model.

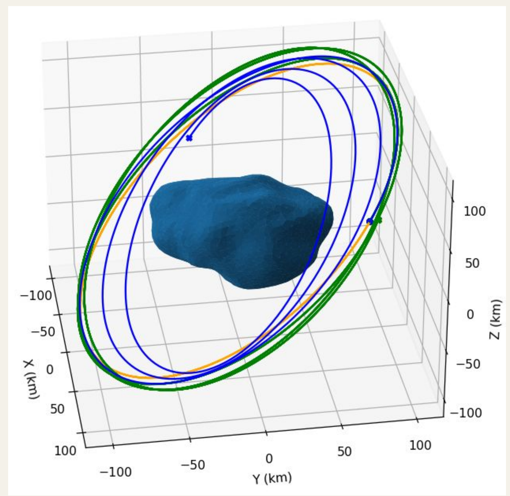

Figure 5: Shape model of asteroid Lutetia used for simulation. Orange trajectory: point-mass model; Blue trajectory: polyhedron model; Green trajectories: Spherical harmonics with different n.

Using Lutetia, a less spherical body than Bennu, as subject, the following results were obtained:

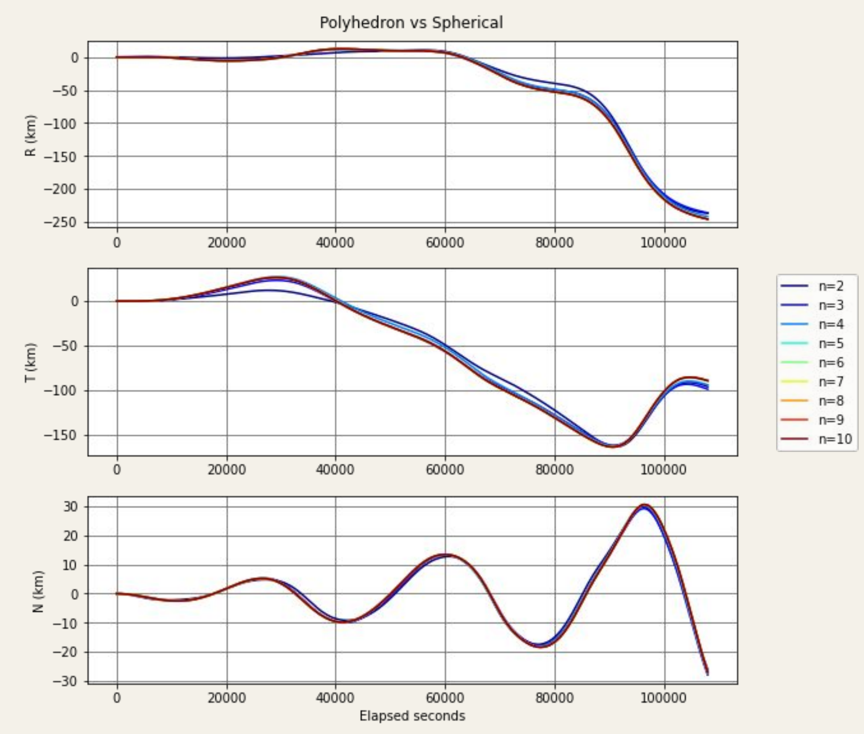

Figure 6: Radial, tangential, and normal component errors for different spherical harmonics models w.r.t. polyhedral model for Lutetia. *sma*=150 km (2.32 Lutetia radii), ecc=0.1, inc=45º

Even though there is a clear convergence on the trajectories as the order of the spherical harmonics model goes up, this convergence is far from the polyhedral model, which shows that shape is a clear limitation for the algorithm.

When analysing the effect of the orbital distance (semi-major axis), the following was observed:

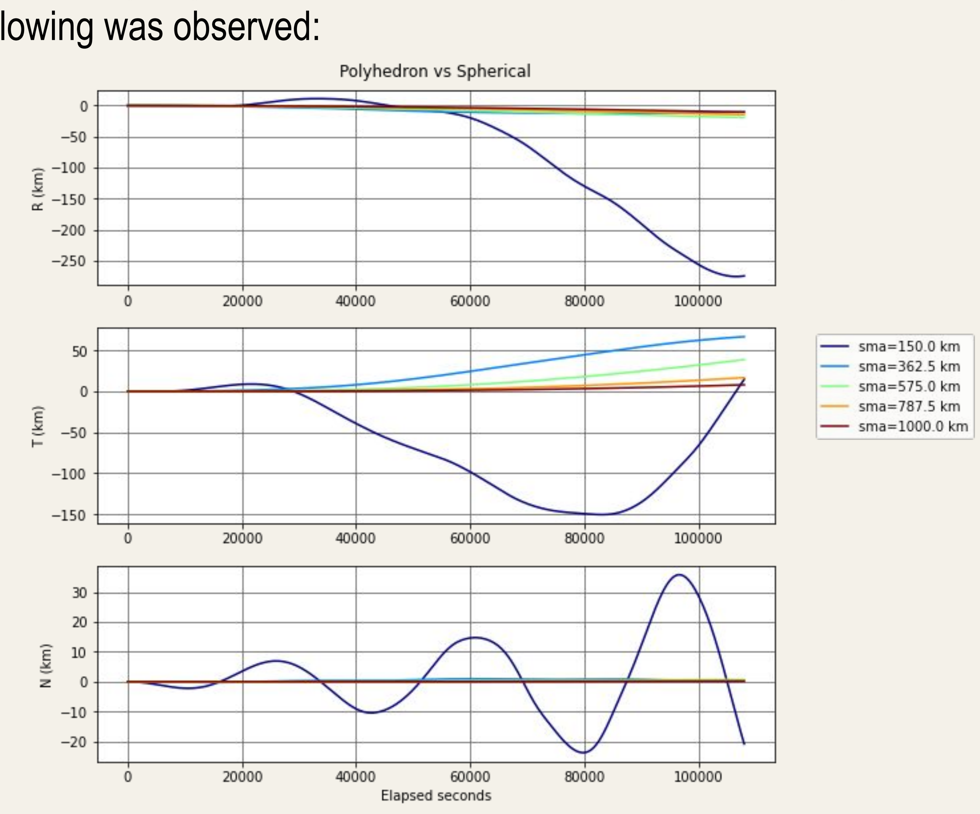

Figure 7: Radial, tangential, and normal component errors for different semi-major axis w.r.t. polyhedral model for Lutetia.

When distance from the body grows larger, the significance of its irregular shape decays until it becomes no longer noticeable. This analysis serves to indicate up to which point these models can be used depending on the gravitational environment.

Conclusion

A tool has been developed to obtain spherical harmonics coefficients from polyhedron shape models. The coefficients computed have been validated using semi-analytical methods.

The further usage of these coefficients in an orbital propagator is assessed by comparing the integrated trajectories with the ones obtained using polyhedron dynamics.

The results show that, in terms of computational effort, the spherical harmonics implementation is far superior to the polyhedron dynamics implementation.

Accuracy-wise, for more spherical bodies, the trajectories converge to the ground-truth. However, this method finds it difficult to replicate the actual gravitational accelerations when orbiting too close to a highly non-spherical body.