Mass lumping scheme for IGA dynamics

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The aim of this study is to provide a new mass lumping scheme for explicit dynamic calculations in isogeometric analysis (IGA).

Shape functions for IGA methods are taken from the Computer-Aided design (CAD) model. The use of higher-order polynomials, usually Non-Uniform Rational B-Splines (NURBS), ensures an exact geometry description [1]. Thus, in comparison to standard finite element method (FEM) the number of elements and therefore the computational costs can be lowered. In addition to that the convergence rate raises equally with the polynomial order. Increasing the degree of shape functions, less elements are necessary for results of the same quality.

In explicit dynamics, mass lumping schemes are commonly applied to reduce computational costs by using diagonal mass matrices. Well-known techniques like row-sum lumping or diagonal scaling method were developed for dynamic analysis with standard FEM. Unfortunately, they are neither suitable for higher-order shape functions in IGA, nor in the spectral element method [2]. It is not possible to take advantage of increasing the polynomial order to lower the number of elements in explicit dynamic IGA, because the error caused by lumping the mass matrix also increases. Thus, other mass lumping schemes, suitable for higher-order shape functions, have to be developed.

As they are already used for isogeometric mortar method [3], several types of dual basis functions (duals) are studied for a new mass lumping scheme. Using duals as test functions in IGA leads to diagonal consistent mass matrices [4] or banded matrices, depending on the chosen type of duals. In case of the banded matrices, additional row-sum lumping causes smaller errors than lumping the original mass matrix with standard NURBS as test functions.

For IGA dynamics with explicit time integration, e.g. the Central Difference Method, using duals as test functions is a very promising approach. They can be easily implemented in existing methods through multiplying the already assembled system matrices with a transformation matrix based on the underlying NURBS curve. Applying the dual lumping scheme, an explicit dynamic analysis could also be performed efficiently using IGA methods with higher-order shape functions.

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