## Adhesion of a cylindrical punch with elastic properties that vary spatially in a gradual manner

Attila Kossa<sup>1,2,3</sup>, René Hensel<sup>4</sup>, Robert M. McMeeking<sup>2,3,4,5</sup>

 <sup>1</sup>Department of Applied Mechanics, Faculty of Mechanical Engineering, Budapest University of Technology and Economics, H-1111 Budapest, Muegyetem rkp. 5., Hungary
 <sup>2</sup>Materials Department, University of California, Santa Barbara CA 93106, USA
 <sup>3</sup>Mechanical Engineering Department, University of California, Santa Barbara CA 93106, USA
 <sup>4</sup>INM – Leibniz Institute for New Materials, Campus D2 2, 66123 Saarbrücken, Germany
 <sup>5</sup>School of Engineering, Aberdeen University, King's College, Aberdeen AB24 3UE, Scotland E-mail: kossa@mm.bme.hu, rmcm@engineering.ucsb.edu

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The stress distribution along the interface between a cylindrical elastic punch and a rigid substrate is analysed. The loading is the tensile stress  $\overline{\sigma}$  along the free end of the punch, as illustrated in Fig.1. (A). Ideal no-slip contact condition is used along the interface. The elastic modulus of the punch varies radially according to the linear relation

$$E(r) = E_0 + E_0 \left(\frac{1}{\beta} - 1\right) \frac{r}{a'},$$
(1)

where  $E_0$  denotes the elastic modulus around the perimeter and  $E_0/\beta$  is the Young's modulus at the center of the punch. The smaller the parameter  $\beta$ , the higher the elastic modulus at the center. The particular case when  $\beta = 1$  is the homogeneous elastic punch. The dimensionless parameter  $\eta = \sigma_z/\overline{\sigma}$  is introduced to measure the ratio of the normal stress along the interface to the external load  $\overline{\sigma}$ . The simulations were performed using UMAT subroutines in Abaqus. Numerical solutions for different  $\beta$  values are shown in Fig.1 (B) using a logarithmic plot. The material is incompressible in the specific case presented here. It was found that the gradually varying elastic modulus along the radial position can be used to significantly reduce the stress magnitudes close to the edge singularity domain, as it can be seen in the figure. These smaller stress values around the perimeter lead to better adhesion strength, which is an essential goal in applications. This report will also present a detailed analysis of the stress distributions for various cases, where radial variation of Poisson's ratio is also considered. These new results can be used to design and fabricate more effective elastic punches for adhesion.



Fig. 1: Schematics of the problem (A); Normalized normal stress distributions along the interface for different values of parameter  $\beta$  (B)

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