Extended Virtual Element Method for fracture problems in two-dimensional linear elasticity

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First proposed in [1], the virtual element method (VEM) is a stabilized Galerkin scheme deriving from mimetic finite differences. The VEM allows for very general polygonal meshes, and does not require the explicit knowledge of the shape functions within the problem domain. For this reason, such functions are dubbed virtual. Instead, the discrete counterpart of the continuum formulation of the problem is defined by means of a suitable projection of the virtual shape functions onto a polynomial space, which allows the decomposition of the bilinear form into a consistent part, reproducing the polynomial space, and a correction term guaranteeing stability. Inspired by the features of the extended finite element method (X-FEM) [2], we proposed an extended virtual element method (X-VEM) for the Laplace problem with singular or discontinuous solutions [3].

In the present contribution, we devise an X-VEM for two-dimensional elastic fracture problems, in which we extend the standard virtual element space with the product of vector-valued virtual nodal shape functions and suitable enrichment fields [4]. We define an extended projection operator that maps functions in the extended virtual element space onto a set spanned by the space of linear polynomials augmented with the enrichment fields. In particular, for the crack tip singularity, we choose, as enrichments, scaled mode I and mode II crack opening displacements fields. Once the element projection matrix has been computed, consistency matrix is obtained as in standard VEM, while the stabilization part must be suitably constructed. We present several numerical examples in 2D elastic fracture to assess convergence and accuracy of the proposed method for both quadrilateral and general polygonal meshes.

References