A REAL OPTIONS APPROACH TO VALUING AGRICULTURAL RESOURCE ASSETS UNDER UNCERTAINTY: US CORN CROPS

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Abstract

Real options are well known for their usefulness in evaluating non-renewable resources. This study derives an asset valuation model using real options methodology to evaluate renewable agriculture investments. The model calculates the value of an investment project as well as computing the critical strike prices at which it becomes optimal to exercise various options over the asset, including when to invest (commence or recommission operations), disinvest (temporarily decommission or delay operations) or abandon the asset altogether. The model incorporates the real options approach into a traditional valuation framework to develop an objective means for calculating a risk-adjusted discount rate applicable to traditional discounted cash flow valuations.

Keywords: Real Options, Renewable resources, Agriculture Valuation

JEL classification: G11, G31 G32

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1. Introduction

This study investigates the evaluation of renewable agricultural resource assets using real options. Whilst initial real options literature focussed on natural resource assets with finite reserves, there is a growing research interest in applications involving renewable resource assets. This is unsurprising given that renewable resources such as water, food and renewable energy, are of vital importance to the world's population. Our research has implications for financial practitioners and policy makers concerned with obtaining accurate and reliable valuations for agricultural investments. For example, the recent Warrnambool Cheese Ltd takeover valuations exceed by tenfold both the market value before the takeover offer and the book value. This research lends itself to practical applications which could assist policy-makers and investors in gaining a better understanding of the effects of uncertainty on investments in renewable agriculture resources as well as the effectiveness of policies relating to taxes and subsidies at a firm level.

Real options approaches to evaluating investment opportunities are heralded as significant theoretical developments in financial economics (Schwartz & Trigeorgis, 2004). Within these approaches, the managerial flexibility to react to new information and the management of uncertainty in future conditions confer significant value to an investment (Trigeorgis & Mason, 1987; Kelly, 2015). Over recent decades these approaches have been applied to evaluate investment decisions across a wide range of assets albeit mainly theoretically-based. While much of the early literature has focussed on natural resource assets which have finite reserves there has been a growing research interest in applications involving renewable resource assets. This study derives an innovative working asset valuation model within the real options methodology to evaluate renewable agricultural resource assets. The model calculates the value of an investment project as well as computing the critical strike prices at which it becomes optimal to exercise various options over the asset, including when to invest (commence or recommission operations), disinvest (temporarily decommission or delay operations) or abandon the asset altogether. An implementation of the renewable resource model including a user-friendly interface is presented. The case of U.S. Corn crops is evaluated to demonstrate the applicability of the model to a real world setting.

The remainder of this paper is as follows. Section 2 provides a theoretical basis for the valuation of agricultural inventory by adapting the theory of natural resources. Section 3 outlines the solution for a renewable agriculture real options model, while section 4 presents the results and discussion of findings for U.S corn crops. Section 5 concludes the paper.

2. Theoretical Model

Evaluating investment under uncertainty is a well-known problem of central importance in financial economics (Cortazar & Schwartz, 1993). Specific issues arise when the valuation of such investments is contingent on the realisation of one or more stochastic variables (Cortazar & Schwartz, 1993; Colwell et al., 2002). While these issues are not unique to any particular industry, they are especially relevant to renewable resource investments. This is due to the value of renewable-resource assets being dependent on the volatile nature of market prices for their underlying commodities.

Traditional DCF method is widely criticised for neglecting the stochastic nature of output prices and its inability to capture value created by the strategic capacity of management to respond operationally to output price volatility over the life of an investment (Brennan and Schwartz, 1985). The shortcomings of the traditional methods, which essentially ignore these strategic concerns, led to the development of alternative methods to value real investments. The best-known of these approaches is the real options pricing approach by arbitrage methods.

Real options are, in a practical sense, analogous to financial options but include various strategic and operational flexibilities that impact on the market value of the asset. A real option exists where a firm or investor reserves various strategic and operational alternatives pertaining to a particular asset which can be activated (or deactivated) under different sets of business conditions. For example, McDonald and Siegel (1986) describe how the investment opportunity is similar to a perpetual call option whereby it gives the investor the right to invest (the exercise price of the option) and receive an asset (the share of a stock). Similarly, Kester (1984) views "growth options' as the discretionary opportunity to invest in productive assets like plant, equipment and brand names with the cost outlay equivalent to the exercise price of a call option. In a broader sense real options may be categorised as being either strategic flexibilities or operating flexibilities (Kelly, 2015).

Real options theory posits that the markets value the rights of firms (or investors) to exercise specific strategic and/or operational options during the course of the business cycle. Furthermore, the more volatile the business conditions, the more valuable are the options that enable the owners of assets to react under uncertainty in order to maximise gains and limit potential losses. This is consistent with a number of empirical studies which have found that options inherent to investment opportunities tend to be valued positively (as measured by a premium) in markets involving real, physical assets (see, for example, Quigg, 1993; Berger, Ofek & Swary, 1996; Kelly, 2015). Accordingly, the real options approach has important practical implications for investment decisions in the sense that it is useful to augment the traditional NPV measure to effectively enhance the decision-making capabilities of firms (Kelly, 1998).

Whilst the Black and Scholes (1973) model is considered the seminal paper for pricing financial options, the first, and generally also considered seminal, paper that significantly defined the area for evaluating real options using continuous-time modelling is Brennan and Schwartz (1985). In this paper they derived a real options pricing model and further illustrated its use via a manufactured copper mine example and a contrived infinite resource mine. In the spirit of the work of Black and Scholes, Brennan and Schwartz (1985) develop a system of partial differential equations whereby the value of the mine's cash flows is dependent on the volatility of output prices, the respective operational policy and the output production rate.

During the same period, other theoretical developments in real options valuation models include the work of McDonald and Siegel (1985) which developed a similar methodology for valuing risky investment projects which reserve the option to shut down production temporarily and without cost whenever variable costs exceed operating revenues. McDonald and Siegel (1986) and Madj and Pindyck (1989) consider the use of the investment rate as the control variable instead of the production rate, the former investigating the value of the option to wait to invest, while the latter applies option pricing methods to derive optimal decision rules for investment outlays over the entire construction program.

It is well-known that empirical research has indeed lagged the development of conceptual frameworks of real options theory (Colwell et al., 2003; Kelly, 2015). That is, although a good deal of work has been developed with respect to conceptual models that treat various aspects of real options from a theoretical perspective, attempts to empirically test the validity of the value of optionality in a real-world setting remain sparse. In the attempts that have been made, a significant portion of this research has been applied to specific cases involving natural resource assets. Commodity markets, in particular, are suitable case studies due to the fact that they are typically highly irreversible investments which exhibit significant underlying spot price volatility (thus the problem is especially acute for valuing these particular assets) and have well developed futures markets (Cortazar, Gravet & Urzua, 2005). However, the typical private and confidential nature of a firm's financial and operational records does indeed make it difficult to compile a relevant and complete set of data to conduct this type of analysis (Colwell et al., 2003).

In a classic empirical study by Quigg (1993) focussed on option-based land valuations. The study searched for empirical evidence for the value of the option to wait to develop land parcels by examining market prices of 2700 land transactions in Seattle during the period 1976-1979. In particular, the study found that market prices reflect a premium for the option to wait to be on average 6% of the land value. This work suggests that investors appear either to directly apply a real option valuation framework, or otherwise, they behave in a manner consistent with predictions of real options theory. Berger et al. (1996) researched how firms value the option to abandon an asset. Their approach was to investigate whether investors use balance sheet information to value their option to abandon the continuing business at the exit value of the asset. They treated the option to abandon as analogous to owning an insurance policy that pays off if the firm performs below expectations. Their method was to analyse the relationship between book value and exit value for major asset classes by examining the discontinued operations footnotes of 157 firms. They found a positive, highly significant relationship between market and exit values and that, if all other factors are equal, investors prefer firms with greater exit values over those with lower values.

Renewable resources are those natural resources useful to human economies that exhibit growth, maintenance, and recovery from exploitation over an economic horizon (Erickson, 2002). While some renewable resources involve a rotational period subject to growth and maintenance (such as forestry resources, seasonal orchids and live fish stocks harvesting) others (for example, energy derived from solar and wind farms as well as water catchments and desalination utilities) are subject to regular, ongoing recovery to inventories. Literature involving applications of real options approaches to evaluate renewable resource investments tends to be growing, particularly with respect to such areas as agriculture, forestry and renewable energy.

Examples of real options applications to agricultural investments includes soybean processing (Plato, 2001), tart cherry production (Nyambane & Black, 2004), dairy operations (Engel & Hyde, 2003; Tauer, 2006), citrus operations (Iwai, Emerson and Roka, 2009) and livestock farms (Bartolini, Gallerani & Viaggi, 2010). In particular, Tauer (2006) uses the analytic approach of Dixit (1989), which is close in spirit to the Brennan and Schwartz (1985) model, to calculate the entry and exit prices for New York dairy farming. The study found there was a wide range between exit and entry prices for individual farmers which was affected by the relative size of the farm and that smaller farms required higher prices to induce entry and would exit at higher prices also.

A real options application to forestry investment, which spans several decades, has predominantly examined the investment timing options involving the underlying timber resource. These include early works by Clarke and Reed (1987) who derive an analytical model which determines economically optimal harvesting policies under both single and rotational conditions, and Morck, Schwartz and Strangeland (1989) who develop a model to evaluate a white pine forestry asset in Canada which holds a ten-year logging lease. Morck et al. (1989) use the cutting rate as the optimal control whereby both the timber price and level of inventories are governed by stochastic processes that follow geometric Brownian motion. The authors acknowledge that their model is equally applicable to fisheries resources and any other similar renewable resource management problem.

Investment in forestry assets is also examined by Duku-Kaakyire and Nanang (2002) who evaluate a set of managerial options using the binomial option pricing method. These options include the timing option to delay reforestation, the option to expand the size of the wood processing plant, the option to abandon the processing plant if timber prices fall below a certain threshold or due to corporate takeover, and multiple options in which all three options are evaluated together. While others such as Kerr, Martin, Kimura, Perera and Lima (2009) analyse the optimal timing decision of when to harvest a stand of eucalyptus trees in Brazil.

Recent studies involving renewable energy investments include Schmit, Luo and Tauer (2008) who evaluate the optimal trigger prices for exercising the options of entering, suspending, reactivating and exiting a corn-based, dry-grind ethanol facility. They note that while research into ethanol firm operations have been analysed from the perspective of NPV, return on investment and break-even analysis, however, little attention has been paid to evaluating these optimal investment decisions from a real options perspective. By analysing the gross margin of ethanol price over corn price they find that, compared to a standard NPV analysis, option values increase entry prices and lower exit prices.

The option to switch between alternative sources of production using renewable energy is analysed by Kjærland and Larsen (2009). They apply the switching model developed by Kulatilaka (1988) to evaluate the operational flexibility relating to a hydro-based operator who holds the option to add thermal power to augment production. This switching option creates value for a hydro operator as it allows the operator to reschedule more production during peak price periods. The study finds that significant option values arise when thermal power plants are controlled by a hydro operator.

The most well-known of the options pricing approaches is the binomial lattice method. A significant operational advantage of using this approach, and the real options approach more generally, is that it does not require an estimated value of the expected cash flows based on forecasted output price movements in the underlying asset. In applying the binomial method, the expected future price movements are calculated using the current output price and the volatility of output prices which is inferred from an historical distribution.

Other option pricing techniques, such as the continuous-time models, are the most operationally complex of the real options models. The complexity in using these techniques is due to the need to implement various numerical methods to solve the resultant set of differential equations that these models are cast as (see, for example, Brennan & Schwartz, 1985). The method generally arrives at these differential equations after careful consideration and application of the mathematics of a standard Gauss-Wiener process in conjunction with Ito's Lemma to derive the total differential of a function of stochastic variables. The resulting set of differential equations establish a continuous-time arbitrage condition which describes the value of an investment based upon the current spot price, the resource inventory and a number of other model parameters. However these equations are rarely amenable to analytic, or closed-form, solution and hence they require unique implementations of numerical methods tuned specifically to cover the applications in real options cases.

A continuous-time model to evaluate real options involving natural resource assets was published by Brennan and Schwartz (1985). This model calculates the present value of the cash flows of an asset based on a no arbitrage argument between the investment opportunity itself and a self-financing portfolio of risk-free bonds and future contracts over the underlying commodity (Kelly, 2015). This seminal paper demonstrated a stylised application of this approach to a copper mine example where the only source of uncertainty is the output price of the underlying commodity.

The Brennan and Schwartz (1985) model assumes that the value of the asset is a function of the commodity price and the quantity of reserves. This model assumes that the only source of uncertainty is the future volatility in the price of the underlying commodity (Kelly, 2015). The spot price of an underlying commodity is modelled as a continuous stochastic process in which the logarithm of the randomly changing quantity is assumed to follow standard geometric Brownian motion. Thus the spot price is assumed to be determined competitively and is given by the exogenous stochastic process (Brennan and Schwartz, 1985):

$$\frac{dS}{S} = \mu dt + \sigma dz$$

where

μ is the local trend (drift) in the price,

dz is an increment to a standard Gauss-Wiener process, and,

σ is the instantaneous standard deviation of the spot price.

The first term in this stochastic process captures the expected value (drift) of the change in price. The second term captures the deviation away from this expected value at any point in time due to the random walk that the price is assumed to follow.

The Brennan and Schwartz (1985) model values the mine as a function of the commodity price and quantity of reserves and the only source of uncertainty is the future volatility in the returns on the underlying asset. The equations specifying this model allows the value of an open and operating mine, $v(s,Q)$, and the value of a closed (temporarily shut down) mine, $w(s,Q)$, to be calculated. Under these conditions, Brennan and Schwartz (1985) specify the deflated1 value of a mine as follows:

Let $v(s,Q)$ be the value of the open mine and $w(s,Q)$ be the value of the closed mine. The Brennan and Schwartz (1985) equations that must be solved to find v and w are given by:

$$\max_{q \in (\underline{q}, \bar{q})} [\frac{1}{2}\sigma^2 s^2 v_{ss} + (r-\kappa)sv_s - qv_Q + q(s-a) - \tau - (r+\lambda_1)v] = 0$$

$$\frac{1}{2}\sigma^2 s^2 w_{ss} + (r-\kappa)sw_s - f - (r+\lambda_0)w = 0$$

where

$$\tau = t_1 qs + \max\,[t_2 q[s(1-t_1)-a],0]$$

$r = \rho - \pi$ is the real interest rate

subject to the boundary conditions:

$$w(s_0^*,Q) = 0$$

$$v(s_1^*,Q) = \max[w(s_1^*,Q) - k_1(Q),0]$$

$$w(s_2^*,Q) = v(s_2^*,Q) - k_2(Q)$$

$$w(s,0) = v(s,0) = 0$$

$$w_s(s_0^*,Q) = 0;$$

$$v_s(s_1^*,Q) = \begin{cases} w_s(s_1^*,Q) & if \quad w(s_1^*,Q) - k_1(Q) \geq 0, \\ 0 & if \quad w(s_1^*,Q) - k_1(Q) < 0; \end{cases}$$

$$w_s(s_2^*,Q) = v_s(s_2^*,Q).$$

A solution to the Brennan and Schwartz (1985) equations determine the optimal policy for opening, closing and abandoning the mine and sets the optimal output rate. Note that, referring to the equation for the open mine, $v(s,Q)$, the optimisation involving the rate of extraction, q, is not intended to drive the system of partial differential equations. Rather, it can be interpreted as saying that for any one spot price, there is a rate of production, bounded by the minimum and maximum production/capacity constraints, that maximises the value of the mine $v(s,Q)$.

3. Renewable Resource Model

Base on the Brennan and Schwartz Model, More specifically, a new model for evaluating renewable resource investments is derived.

This model is based upon the seminal Brennan and Schwartz (1985) framework, but involves recasting that framework to suit assets that have a renewable inventory, under the assumption that the rate of extraction does not exceed the rate at which the resource is replenished. An integral part of the derivation of the new model is the specification of boundary conditions similar to, albeit mathematically distinct from, the boundary conditions of the original framework. More generally, the new model is derived and analysed through the application of the concept of analytic continuation (Harris, 1992) which involves extending the basic functions and equations of the model, continuously to first derivatives, over the entire phase space of the parameters. It will be shown that the resultant analytically continued model's equations can be solved to produce closed form solutions. Specific solutions can only be calculated via numerical techniques and consequently computational implementations of general solutions for the final model are developed.

The Brennan and Schwartz (1985) model consists of two differential equations which describe the value of a mine based upon the current spot price and a number of other model parameters. The original model for a finite life, diminishing inventory (when being operated) asset is:

$$\max_{q\in(\underline{q},\bar{q})}[\frac{1}{2}\sigma^2 s^2 v_{ss} + (r-\kappa)sv_s - qv_Q + q(s-a) - \tau - (r+\lambda_1)v] = 0 \quad \text{(open mine)}$$

$$\frac{1}{2}\sigma^2 s^2 w_{ss} + (r-\kappa)sw_s - f - (r+\lambda_0)w = 0 \quad \text{(closed mine)}$$

where

$$\tau = t_1 qs + \max\,[t_2 q[s(1-t_1) - a],0]$$

$r = \rho - \pi$ is the real interest rate

subject to the boundary conditions:

$$w(s_0^*,Q) = 0$$

$$v(s_1^*,Q) = \max[w(s_1^*,Q) - k_1(Q),0]$$

$$w(s_2^*,Q) = v(s_2^*,Q) - k_2(Q)$$

$$w(s,0) = v(s,0) = 0$$

$$w_s(s_0^*, Q) = 0;$$

$$v_s(s_1^*, Q) = \begin{cases} w_s(s_1^*, Q) & \text{if} \quad w(s_1^*, Q) - k_1(Q) \geq 0, \\ 0 & \text{if} \quad w(s_1^*, Q) - k_1(Q) < 0; \end{cases}$$

$$w_s(s_2^*, Q) = v_s(s_2^*, Q).$$

The development of a real options model applicable to evaluating renewable resource assets, based on the Brennan and Schwartz theory, involves the use of analytic continuation. Analytic continuation is a well-known technique in the area of complex function analysis mathematics (Harris, 1992). The technique involves extending the domain of an analytic function beyond its original (or defined) range in a manner which keeps that function continuous to first derivative into the extended domain. The practical implications of the use of the method of analytic continuation is that the analytically-continued function can then be used to extend the original function into a new domain where the original function initially defined becomes divergent, is undefined, has no physical meaning, or has non-analytic behaviour or form (see, for example, Harris, 1992).

In this study analytic continuation is used to cover values of the spot price lower than the abandonment option, a physically unreasonable range of the spot price domain. In their original model, Brennan and Schwartz imposed the reasonable constraint that the value of the mine is zero when the spot price is lower than the abandonment option. This causes a non-analyticity in their model (in the first derivative) and is a contributory reason to the generally accepted belief that their model is insoluble analytically.

We produce an analytically continued version of the real options Brennan and Schwartz (1985) functions $v(s,Q)$ and $w(s,Q)$ by allowing the functions to continue smoothly beyond their boundary endpoints and into regions of the spot price phase space where they strictly (according to Brennan & Schwartz, 1985) are not defined. That is, the real options functions $v(s,Q)$ and $w(s,Q)$ are smoothly continued into the lower spot price region $s < s_0^*$ and the upper spot price region of $s > s_2^*$. Achieving an analytic solution to a version of this model provides a more complete coverage of phase space thus furthering conventional insight into the behaviour of the model. Necessarily this requires a reworking of the currently discontinuous boundary conditions specified by Brennan and Schwartz (1985).

The model equations are smooth and continuous over their entire range of spot prices and are not modified, even beyond the physical limits imposed by the boundary conditions. This is in stark contrast to the boundary conditions originally specified which contains embedded discontinuities and discontinuous behaviour. In the case of an asset with a renewable inventory, the Q dependence in the boundary conditions disappears as $Q \to \infty$. Accordingly, the set of physical boundary conditions simplify to:

$$w(s_0) = 0 \qquad (1)$$

$$v(s_1) = w(s_1) - k_1 \tag{2}$$

$$v(s_2) = w(s_2) + k_2 \tag{3}$$

$$v_s(s_1) = w_s(s_1) \tag{4}$$

$$v_s(s_2) = w_s(s_2) \tag{5}$$

Similarly, the discontinuity in the tax function τ is replaced by a smooth tax function thus allowing for a full loss tax off-set (see Brennan and Schwartz, 1985). The tax function thus becomes

$$\tau = t_1 qs + t_2 q(s(1-t_1) - a) \tag{6}$$

Under the above assumptions the (deflated) value of the open mine $v(s,Q)$ (when it is operating at a production rate q which does not exceed the rate of replenishment) satisfies the ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 v}{ds^2} + (r-\kappa)s\frac{dv}{ds} + q(s-a) - t_1 qs - t_2 qs(1-t_1) + t_2 qa - (r+\lambda_1)v = 0 \tag{7}$$

We note that the ODE which values the closed asset remains unchanged from the original specification.

The new model for an infinite resource asset is the following system of equations:

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 v}{ds^2} + (r-\kappa)s\frac{dv}{ds} + q(s-a) - \tau - (r+\lambda_1)v = 0 \tag{8}$$

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 w}{ds^2} + (r-\kappa)s\frac{dw}{ds} - f - (r+\lambda_0)w = 0 \tag{9}$$

Where:

$$\tau = t_1 qs + t_2 q(s(1-t_1) - a)$$

$$r = \rho - \pi$$

Subject to the boundary conditions:

$$w(s_0) = 0$$

$$v(s_1) = w(s_1) - k_1$$

$$v(s_2) = w(s_2) + k_2$$

$$v_s(s_1) = w_s(s_1)$$

$$v_s(s_2) = w_s(s_2)$$

This model has the advantage of being smooth and twice-differentially continuous over the entire spot price phase space. This ensures that conventional solution techniques can now be applied to this system of equations. The equation for the untapped asset can be solved analytically. Solution is readily recognized when the equation is reformulated as a second order ordinary differential Euler equation. That is, equation (9) becomes,

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 w}{dw^2} + (r-\kappa)s\frac{dw}{ds} - f - (r+\lambda_0)w = 0$$

is recast as an Euler Equation:

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 w}{dw^2} + (r-\kappa)s\frac{dw}{ds} - (r+\lambda_0)w = f$$

Thus the value of the closed asset, $w(s)$, is simply the sum of the homogeneous and particular solutions to this Euler equation. At this point it is interesting to note that the actual form of the ODE for the closed asset is identical to the original Brennan and Schwartz (1985). That is, the real difference between the two approaches has no impact on the provision of an analytic solution to this case: an analytic solution is available for both infinite and finite non-operational resource assets.

The homogeneous equation (same equation with a zero RHS) is:

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 w}{ds^2} + (r-\kappa)s\frac{dw}{ds} - (r+\lambda_0)w = 0$$

Following standard techniques, (see, for example, Spencer, Parker, Berry, England, Faulkner, Green, Holden, Middleton & Rogers, 1977a), it is well known that the form of the homogeneous solution will be

$$w(s) = cs^n$$

Where

s is the spot price,

$w(s)$ is the value of the closed asset at a particular spot price, s

c is a constant to be determined by boundary conditions

n is the exponential change in value of the asset with respect to s.

The derivatives of w are:

$$\frac{dw}{ds} = ncs^{n-1}$$

$$\frac{d^2 w}{ds^2} = n(n-1)cs^{n-2}$$

Substituting back into the homogeneous equation gives

$$\frac{1}{2}\sigma^2 s^2 n(n-1)s^{n-2} + (r-\kappa)sns^{n-1} - (r+\lambda_0)s^n = 0$$

$$\Rightarrow \frac{\sigma^2}{2} n(n-1)s^n + (r-\kappa)ns^n - (r+\lambda_0)s^n = 0$$

$$\Rightarrow \left[\frac{\sigma^2}{2} n(n-1) + (r-\kappa)n - (r+\lambda_0)\right]s^n = 0$$

$$\Rightarrow \frac{\sigma^2}{2} n^2 - \frac{\sigma^2}{2} n + (r-\kappa)n - r - \lambda_0 = 0$$

$$\Rightarrow \frac{\sigma^2}{2} n^2 + (r-\kappa-\frac{\sigma^2}{2})n + (-r-\lambda_0) = 0$$

Consequently n is the solution of a quadratic equation and thus determined by the standard quadratic solution formula:

$$n_\pm = \frac{-(r-\kappa-\frac{\sigma^2}{2}) \pm \sqrt{(r-\kappa-\frac{\sigma^2}{2})^2 + 2\sigma^2(r+\lambda_0)}}{\sigma^2}$$

where it is noted that $n_+ > 0$ and $n_- < 0$

Subsequently the full homogeneous solution is given in closed form by

$$w_h(s) = c_+ s^{n_+} + c_- s^{n_-}$$

Imposing the physical conditions that $w(s)$ is a monotonically increasing function of s implies $c_- \equiv 0$, and hence the homogeneous solution to the Euler ODE becomes:

$$w_h(s) = c_+ s^{n_+}$$

By inspection $w_p(s)$ must equal some constant independent of spot price s. That is $\frac{d^2 w}{ds^2} = 0$ and $\frac{dw}{ds} = 0$, and hence:

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 w}{ds^2} + (r-\kappa)s\frac{dw}{ds} - (r+\lambda_0)w = f$$

$$\Rightarrow -(r+\lambda_0)w = f$$

$$\Rightarrow w = -\frac{f}{r+\lambda_0}$$

$$\Rightarrow w_p(s) = -\frac{f}{r+\lambda_0}$$

In practical terms, the particular solution $w_p(s)$ is that component of the closed asset value that is independent of the spot price. This component is thus the maintenance costs of the asset discounted by the appropriate interest and tax rates.

Combining the above, including a notational change of + to 0 in the constants for clarity with the overall model, results in the general, analytic solution to the closed asset to be:

$$w(s) = c_0 s^{n_0} - \frac{f}{r+\lambda_0}$$

where c_0 is an integration constant, and

$$n_0 = \frac{-(r-\kappa-\frac{\sigma^2}{2}) + \sqrt{(r-\kappa-\frac{\sigma^2}{2})^2 + 2\sigma^2(r+\lambda_0)}}{\sigma^2}$$

The value of c is determined by appropriate boundary conditions. At this point it is noted that isolating the closed asset invalidates those conditions relating the values between the open and closed assets. That is, the only valid boundary condition (that does not depend on the value of the closed asset) is

$$w_s(s_0) = 0 \, .$$

We see that this boundary condition requires the value of the spot price at which the value is zero which would imply $c = 0$, a non-physical case, and thus discarded. The solution for an operational asset can be found by rearranging equation (3.08) for the open mine (operating in the presence of an inexhaustible inventory) into an Euler equation:

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 v}{ds^2} + (r-\kappa)s\frac{dv}{ds} + q(s-a) - t_1 qs - t_2 qs(1-t_1) + t_2 qa - (r+\lambda_1)v = 0$$

$$\Rightarrow \frac{1}{2}\sigma^2 s^2 \frac{d^2 v}{ds^2} + (r-\kappa)s\frac{dv}{ds} + qs - qa - t_1 qs - t_2 qs(1-t_1) + t_2 qa - (r+\lambda_1)v = 0$$

$$\Rightarrow \frac{1}{2}\sigma^2 s^2 \frac{d^2 v}{ds^2} + (r-\kappa)s\frac{dv}{ds} + qs(1-t_1) - t_2 qs(1-t_1) - qa + t_2 qa - (r+\lambda_1)v = 0$$

$$\Rightarrow \frac{1}{2}\sigma^2 s^2 \frac{d^2 v}{ds^2} + (r-\kappa)s\frac{dv}{ds} + qs(1-t_1)(1-t_2) - qa(1-t_2) - (r+\lambda_1)v = 0$$

$$\Rightarrow \frac{1}{2}\sigma^2 s^2 \frac{d^2 v}{ds^2} + (r-\kappa)s\frac{dv}{ds} - (r+\lambda_1)v = -qs(1-t_1)(1-t_2) + qa(1-t_2) \tag{10}$$

This Euler equation can be solved using standard techniques to find the solution to the open asset case. The solution is simply the sum of the homogeneous and particular solutions.

The homogeneous solution is the solution of the following ODE:

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 v}{ds^2} + (r-\kappa)s\frac{dv}{ds} - (r+\lambda_1)v = 0 \tag{11}$$

Following standard techniques, (see, for example, Spencer et al., 1977a), the form of the homogeneous solution will be

$$v_H(s) = cs^n$$

where

c is a constant to be determined by boundary conditions

n is the exponential change in value of the asset with respect to s.

The values of c and n are found by substitution. The derivatives of v are:

$$\frac{dv}{ds} = ncs^{n-1}$$

$$\frac{d^2 v}{ds^2} = n(n-1)cs^{n-2}$$

Substituting back into the homogeneous equation (11) and removing c yields

$$\frac{1}{2}\sigma^2 s^2 n(n-1)s^{n-2} + (r-\kappa)sns^{n-1} - (r+\lambda_1)s^n = 0$$

$$\Rightarrow \frac{\sigma^2}{2}n(n-1)s^n + (r-\kappa)ns^n - (r+\lambda_1)s^n = 0$$

$$\Rightarrow \left[\frac{\sigma^2}{2}n(n-1) + (r-\kappa)n - (r+\lambda_1)\right]s^n = 0$$

Therefore either $s^n = 0$ or $\left[\frac{\sigma^2}{2}n(n-1) + (r-\kappa)n - (r+\lambda_1)\right] = 0$

$$\Rightarrow \frac{\sigma^2}{2} n^2 - \frac{\sigma^2}{2} n + (r-\kappa)n - r - \lambda_1 = 0$$

$$\Rightarrow \frac{\sigma^2}{2} n^2 + (r-\kappa-\frac{\sigma^2}{2})n + (-r-\lambda_1) = 0$$

Thus, the solutions for n are given by substitution into the standard quadratic solution formula, leading to:

$$n_\pm = \frac{-(r-\kappa-\frac{\sigma^2}{2}) \pm \sqrt{(r-\kappa-\frac{\sigma^2}{2})^2 + 2\sigma^2(r+\lambda_1)}}{\sigma^2}$$

where it is noted that $n_+ > 0$ and $n_- < 0$ (as was the case for the closed asset).

The homogeneous solution is given by

$$v_H(s) = c_0 s^{n_+} + c_1 s^{n_-}$$

Imposing the physical condition that $v(s)$ remains bound to being no greater than linear in s as $s \to \infty$ implies that $c_0 \equiv 0$, and hence

$$v_H(s) = c_1 s^{n_1} \tag{12}$$

where

$$n_1 = \frac{-(r-\kappa-\frac{\sigma^2}{2}) - \sqrt{(r-\kappa-\frac{\sigma^2}{2})^2 + 2\sigma^2(r+\lambda_1)}}{\sigma^2}$$

By inspection of (10) $v_p(s)$ must equal some function that is linearly dependent on spot price s. That is, $v_p(s)$ can be written as:

$$v_P(s) = As + B$$

where A and B are constants. Thus, the derivatives with respect to spot price, s, will be

$$\frac{dv}{ds} = A$$

and

$$\frac{d^2v}{ds^2} = 0$$

Substituting this back into the Euler equation (10) yields

$$\rightarrow (r-\kappa)sA - (r+\lambda_1)(As+B) = -qs(1-t_1)(1-t_2) + qa(1-t_2)$$

$$\Rightarrow (r-\kappa)sA - As(r+\lambda_1) - B(r+\lambda_1) = -qs(1-t_1)(1-t_2) + qa(1-t_2)$$

$$\Rightarrow (r-\kappa-r-\lambda_1)sA - B(r+\lambda_1) = -qs(1-t_1)(1-t_2) + qa(1-t_2)$$

$$\Rightarrow -A(\kappa+\lambda_1)s - B(r+\lambda_1) = -qs(1-t_1)(1-t_2) + qa(1-t_2)$$

Grouping according to common terms in s yields:

$$-A(\kappa+\lambda_1) = -q(1-t_1)(1-t_2)$$

$$\Rightarrow A = \frac{q(1-t_1)(1-t_2)}{(\kappa+\lambda_1)}$$

and

$$-B(r+\lambda_1) = qa(1-t_2)$$

$$\Rightarrow B = -\frac{qa(1-t_2)}{(r+\lambda_1)}$$

Consequently, the particular solution to the open asset for the infinite resource case is found to be:

$$v_P(s) = \frac{q(1-t_1)(1-t_2)}{(\kappa+\lambda_1)}s - \frac{qa(1-t_2)}{(r+\lambda_1)} \tag{13}$$

Combining the above two solutions (12) and (13) yields the full general solution to the open asset for the case of an infinite resource

$$v(s) = c_1 s^{n_1} + \frac{q(1-t_1)(1-t_2)}{(\kappa+\lambda_1)}s - \frac{qa(1-t_2)}{(r+\lambda_1)} \tag{14}$$

where the constant of integration, c_1, is found from the boundary conditions and

$$n_1 = \frac{-(r-\kappa-\frac{\sigma^2}{2}) - \sqrt{(r-\kappa-\frac{\sigma^2}{2})^2 + 2\sigma^2(r+\lambda_1)}}{\sigma^2}$$

The general solution to the renewable resource asset model is thus the combination of the above 2 solutions, one case being for when the asset (being modelled) is closed and the other for when it is operational. That is, the general solution is:

$$w(s) = c_0 s^{n_0} - \frac{f}{r+\lambda_0}$$

$$v(s) = c_1 s^{n_1} + \frac{q(1-t_1)(1-t_2)}{(\kappa+\lambda_1)}s - \frac{qa(1-t_2)}{(r+\lambda_1)}$$

To determine the various constants it is necessary to apply the boundary conditions listed as (1) to (5). The first of these boundary conditions (1) uniquely specifies the value of s_0 only. This then leaves 4 equations in 4 unknowns, being c_0, c_1, s_1 and s_2. Equations $v_s(s_1) = w_s(s_1)$, and $v_s(s_2) = w_s(s_2)$ require first derivatives as follows:

$$\frac{dv}{ds} = c_1 n_1 s^{n_1 - 1} + \frac{q(1-t_1)(1-t_2)}{(\kappa + \lambda_1)} \tag{15}$$

and

$$\frac{dw}{ds} = c_0 n_0 s^{n_0 - 1} \tag{16}$$

Substituting equations (7), (14), (15) and (16) into the boundary conditions (2) – (5) yields the standard problem of solving the following 4 simultaneous equations to determine the unknown constants c_0, c_1, s_1 and s_2:

$$c_1 s_1^{n_1} + \frac{q(1-t_1)(1-t_2)}{(\kappa + \lambda_1)} s_1 - \frac{qa(1-t_2)}{(r + \lambda_1)} = c_0 s_1^{n_0} - \frac{f}{r + \lambda_0} - k_1$$

$$c_1 s_2^{n_1} + \frac{q(1-t_1)(1-t_2)}{(\kappa + \lambda_1)} s_2 - \frac{qa(1-t_2)}{(r + \lambda_1)} = c_0 s_2^{n_0} - \frac{f}{r + \lambda_0} + k_2$$

$$c_1 n_1 s_1^{n_1 - 1} + \frac{q(1-t_1)(1-t_2)}{(\kappa + \lambda_1)} = c_0 n_0 s_1^{n_0 - 1}$$

$$c_1 n_1 s_2^{n_1 - 1} + \frac{q(1-t_1)(1-t_2)}{(\kappa + \lambda_1)} = c_0 n_0 s_2^{n_0 - 1}$$

The solution of these 4 equations is made difficult due to their explicit non-linearity. For brevity define the following constants:

$$\frac{q(1-t_1)(1-t_2)}{(\kappa + \lambda_1)} \equiv p$$

and

$$\frac{qa(1-t_2)}{(r + \lambda_1)} \equiv m$$

Thus the system of non-linear equations to be solved simultaneously becomes:

$$c_1 s_1^{n_1} + p s_1 - m = c_0 s_1^{n_0} - \frac{f}{r + \lambda_0} - k_1 \tag{17}$$

$$c_1 s_2^{\ n_1} + p s_2 - m = c_0 s_2^{\ n_0} - \frac{f}{r+\lambda_0} + k_2 \tag{18}$$

$$c_1 n_1 s_1^{\ n_1 - 1} + p = c_0 n_0 s_1^{\ n_0 - 1} \tag{19}$$

$$c_1 n_1 s_2^{\ n_1 - 1} + p = c_0 n_0 s_2^{\ n_0 - 1} \tag{20}$$

By inspection, these equations are linear in c_0 and c_1. An exact solution can be readily formulated from any 2 of the above equations, for these 2 unknowns. The trick is to choose the 2 equations for c_0 and c_1 such that the "simpler" equations are left to solve for s_1 and s_2. Thus it is proposed that the first 2 equations (17) and (18) are solved for c_0 and c_1, and accordingly, (19) and (20) are solved for s_1 and s_2.

Explicitly solving for c_0 and c_1, equations (17) and (18) then yield:

$$c_1 s_1^{\ n_1} + p s_1 - m = c_0 s_1^{\ n_0} - \frac{f}{r+\lambda_0} - k_1$$

$$\Rightarrow c_1 s_1^{\ n_1} = c_0 s_1^{\ n_0} - \frac{f}{r+\lambda_0} - k_1 + m - p s_1$$

$$\Rightarrow c_1 = \left(c_0 s_1^{\ n_0} - \frac{f}{r+\lambda_0} - k_1 + m - p s_1 \right) s_1^{\ -n_1} \tag{21}$$

Substituting (21) back into equation (20) yields:

$$c_1 s_2^{\ n_1} + p s_2 - m = c_0 s_2^{\ n_0} - \frac{f}{r+\lambda_0} + k_2$$

$$\Rightarrow \left(c_0 s_1^{\ n_0} - \frac{f}{r+\lambda_0} - k_1 + m - p s_1 \right) s_1^{\ -n_1} s_2^{\ n_1} + p s_2 - m = c_0 s_2^{\ n_0} - \frac{f}{r+\lambda_0} + k_2$$

$$\Rightarrow \left(c_0 s_1^{\ n_0} - \frac{f}{r+\lambda_0} - k_1 + m - p s_1 \right) s_2^{\ n_1} + p s_2 s_1^{\ n_1} - m s_1^{\ n_1} = c_0 s_2^{\ n_0} s_1^{\ n_1} - \frac{f s_1^{\ n_1}}{r+\lambda_0} + k_2 s_1^{\ n_1}$$

$$\Rightarrow c_0 s_1^{\ n_0} s_2^{\ n_1} - c_0 s_2^{\ n_0} s_1^{\ n_1} = \frac{f s_2^{\ n_1}}{r+\lambda_0} + k_1 s_2^{\ n_1} - m s_2^{\ n_1} + p s_1 s_2^{\ n_1} - p s_2 s_1^{\ n_1} + m s_1^{\ n_1} - \frac{f s_1^{\ n_1}}{r+\lambda_0} + k_2 s_1^{\ n_1}$$

$$\Rightarrow c_0 (s_1^{\ n_0} s_2^{\ n_1} - s_2^{\ n_0} s_1^{\ n_1}) = \frac{f (s_2^{\ n_1} - s_1^{\ n_1})}{r+\lambda_0} + k_1 s_2^{\ n_1} + k_2 s_1^{\ n_1} + p(s_1 s_2^{\ n_1} - s_2 s_1^{\ n_1}) + m(s_1^{\ n_1} - s_2^{\ n_1})$$

$$\Rightarrow c_0 = \frac{\dfrac{f(s_2{}^{n_1} - s_1{}^{n_1})}{r+\lambda_0} + k_1 s_2{}^{n_1} + k_2 s_1{}^{n_1} + p(s_1 s_2{}^{n_1} - s_2 s_1{}^{n_1}) + m(s_1{}^{n_1} - s_2{}^{n_1})}{(s_1{}^{n_0} s_2{}^{n_1} - s_2{}^{n_0} s_1{}^{n_1})} \quad (22)$$

Thus equations (22) and (21) provide an exact formula for c_0 and c_1 for any given estimates to s_1 and s_2. Equations (19) and (20) are clearly non-linear in s_1 and s_2. Thus analytic formulae for s_1 and s_2 are not possible. Consequently, generating a solution via an appropriately stable numerical technique is the only way to determine s_1 and s_2. Given the non-linearity of equations (19) and (20), iterative techniques such as the Gauss-Seidel (see, for example, Spenser et al., 1977a) or the Newton-Raphson methods (see, for example, Spencer et al., 1977b) are appropriate.

The solution to equations (17) to (20) was achieved by setting up a Gauss-Seidel style iteration around equations (21), (22) and (23) in which, at each step in that sequence, the following calculations were performed:

1. the values of c_0 and c_1 were determined by (21) and (22)
2. these values were then substituted to find s_1 and s_2 via equation (23)
3. The non-linear system of equations (17) to (20) was implemented in a Delphi program.

Figure 1 below displays the boot-up screen and explains the overall layout of the implementation.

Insert Figure 1 here

As shown in Figure 1, the screen design layout consists of a separation of the input and output parameters in the left- and right-hand sides of the screen respectfully. The data input parameters that are listed on the left-hand side of the screen are grouped into several sections including the operational costs and other input parameters used to evaluate the asset. Once the data parameters have been input the next step is to initialise the data values in the output table by clicking the "Initialise all data values in the output tables" button on the right-hand side of the screen. Once these values are set, clicking the "Iterate Gauss-Sedel system until converged" button starts the iteration process.

The system was validated using the published data from Brennan and Schwartz (1985) copper mine example presented in Figure 1. The entire system of equations required 6 iterations of the Gauss-Seidel loop to achieve 8 significant digits of accuracy in each of the 5 variables being calculated.

3. Application to Corn-Crop Investments under Uncertainty

The model is applied to price cropland assets across the U.S. Corn Belt as well as to outline the appropriate operational policy that farmers should follow to manage these assets optimally. Whereas some previous related studies have examined entry and exit decisions regarding new investments in agricultural assets (see, for example, Tauer (2006)), this research evaluates the managerial options available to a particular perennial agricultural operation whereby the initial decision to invest has already been made.

Our model has practical implications which may assist farmers to make optimal decisions regarding whether or not to plant their field crops for the current season and at which price it is worth considering alternative uses for their crop land such as rotating their crops or converting their land for other industrial uses. These managerial decisions have significant implications for valuing agricultural assets that are difficult to incorporate into DCF valuations.

For instance, following the Brennan and Schwartz (1985) example of a copper mine, should the price of copper fall to a sufficiently low level (the critical price S1), then the mine owner should cease operating the mine and incur the costs of closure and annual maintenance on the mine. A farmer faces a similar dilemma as the resource miner regarding their decisions to choose the appropriate operational policy that optimises the value of their investment. Each year farmers must consider whether or not the current spot price is appropriate to warrant exercising the option to outlay the costs to plant their fields for the coming season. Thus for this case study the option associated with the price S1 is interpreted as one where the farmer should forego planting for the current season whilst maintaining the crop land until prices improve and reach the critical price S2. At the critical price S2, it is appropriate to outlay the costs of production to plant the crop. If, however, the option to delay planting has already been exercised, and the price should continue to fall further to the critical price S0, then the farmer should allow the land to lay fallow whilst considering alternative uses for their cropland.

We selected corn as a suitable agricultural commodity to illustrate the application of the model for several reasons. First, corn is the largest component of the world's coarse grain trade and is therefore an economically important soft commodity that is cultivated for a variety of purposes including food, feed and industrial uses (United States Department of Agriculture (USDA), 2011a). Second, since corn is a perennial crop (as such its inventory may be harvested and subsequently replenished on an annual basis), and has well-developed markets, it is deemed appropriate to fit with various assumptions underpinning the model's applicability to a real-world setting. Moreover, if the tenancy of the farm is free from constraints imposed by any specific lease period the investment horizon can be assumed to be infinite (thus satisfying a key assumption of the model). Third, a real options analysis is appropriate for assets such as perennial crops because the farmer holds various operational options over these assets which, as previously discussed, may be exercised at the farmer's discretion.

Accordingly, we focused the case study application of the renewable resource model to evaluate hypothetical investments in corn production assets across the Heartland States of Illinois, Indiana, Iowa, Missouri and Ohio. Valuations are calculated based on cropland consisting of 250 and 1000 acres for which the total area is assumed amenable to crop production. Parameters used in the study are average values calculated using data sourced from various U.S. Government agencies over the 10-year period 2000 to 2009. The parameters used in this study are estimated using USDA census data.

The model requires the variance of the corn price to be calculated. A key assumption of the renewable resource options model is that the prices of the underlying commodity follow a random walk and are log-normally distributed . Thus, using monthly U.S. corn prices data from January

2000 to December 2009, the statistic $d_t = \ln(\frac{s_{t+1}}{s_t})$ is calculated and annualised by multiplying by 12. This resulted in an annually adjusted mean (d_t) of 0.0057 and standard deviation of 0.1891, hence a variance of 0.0358.

Whether the long-term average price of corn is stationary was tested using a Dickey-Fuller test. Using USDA data for average monthly corn prices received by U.S. farmers over the period January 1975 to December 2010 yields 420 observations. The null hypothesis that the corn price is stationary could not be rejected at the 95% confidence.

The model requires estimates for the average rates of production and the associated costs per bushel of corn. The USDA publishes a detailed breakdown of cost data consisting of both the variable and fixed cost components of corn farm production compiled on a regionalised basis. The latest estimates for average total economic costs reported by the USDA for the Heartland Region are for the year 2009, which is $555.15 per planted acre (USDA 2011c). The average production costs for each of the states is calculated by dividing the total costs by the average yields reported for each state over the decade 2000 to 2009 (USDA 2011d). Calculations for the annual rates of production and the resulting production costs per bushel of corn are presented below in Table 1.

The results of these calculations show that farmers with corn crops located in the Heartland States of Iowa and Illinois are relatively low-cost producers, while farmers in Missouri have relatively high costs of production. It is not surprising that farms located in the States of Iowa and Illinois typically produce in excess of one-third of the total U.S corn crop (USDA, 2011a).

Insert Table 1 here

Since the actual risk-free rate is not readily observable (and thus known) the standard approach in the literature is to use the yield on U.S. Treasury securities as a proxy for the risk-free rate. The 10-year U.S. Treasury bond rate was chosen for use in this study.

A key assumption of the model is that the future price of a commodity is a function of the spot price and time to maturity. Because the model assumes the net convenience yield to be proportional to the price it may be estimated directly from the following relationship:

$$F(T) = S(t)e^{(r-\kappa)(T-t)}$$

where F(T) is the futures price, S(t) is the spot price, r is the nominal interest rate and κ is the convenience yield. Thus monthly convenience yields are calculated2 and annualised following the approach adopted by Gibson and Schwartz (1990):

[2] The log changes in the corn price were calculated using average monthly prices received by U.S. farmers published by the USDA.

$$\kappa = r - \frac{1}{\Delta t} * \ln\left[\frac{F(T)}{S(t)}\right]$$

where Δt is equal to 1/12.

The average annualised convenience yield over the period January 2000 to December 2009 was calculated to be 4.39%. The results of these calculations are tabled in Table 2 below.

Insert Table 2 here

The final estimates for the analysis are estimates for federal income taxes, state property taxes and the royalty rate and maintenance costs, costs of closure and costs of reinstating operations. According to Durst (2009), 99% of U.S. farm income is taxed at the individual tax rate rather than the corporate tax rate since most farms are operated as sole proprietors, partnerships, or small business corporations. For this study we apply a tax rate of 15% which is reported by the USDA as the average effective Federal income tax rate for 2004. The average state property tax rates for each state is taken from a 2007 article in The New York Times: Illinois (1.79%), Indiana (2.12%), Iowa (2.15%), Missouri (1.42%) and Ohio (1.81%) . Royalty rates do not apply and are set to zero percent.

Costs for cropland that is currently being maintained are estimated by including only those recurring annual costs of the farm published by the USDA for the Heartland Region. Our estimates include non-real estate taxes and insurance and general farm overhead. These costs are reported by the USDA to total $24 per planted acre for 2009. Costs associated with exercising the option to temporarily delay operations, k_1, may include the costs of storing machinery (and silage if appropriate), paying out relevant redundancies and termination payments on current contracts, as well as any outstanding general liabilities. While the costs to exercise the option to reinstate operations, k_2, may involve hiring and training costs and other miscellaneous recommissioning costs due on the farm. For this study we assume the costs of transitioning between each operating state to be $10,000 each. Costs of abandoning the crop to let the land lie fallow are assumed to be zero.

Having now calculated all of the required parameters for use in the model, a summary of the parameters are detailed in the following Table 3.

Insert Table 3 here

For cropland that is currently being operated, its value, v(s), is defined by the following differential equation

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 v}{ds^2} + (r-\kappa)s\frac{dv}{ds} + q(s-a) - \tau - (r+\lambda_1)v = 0$$

For cropland that is currently not being operated but is rather being maintained until prices improve, its value, w(s), is defined by the differential equation

$$\frac{1}{2}\sigma^2 s^2 \frac{d^2 w}{ds^2} + (r-\kappa)s\frac{dw}{ds} - f - (r+\lambda_0)w = 0$$

The model is subject to the following boundary conditions that determine the optimal operating policy (considering the costs to transition between operating states) in addition to ensuring smoothness over the transition points.

$$w(s_0) = 0$$

$$v(s_1) = w(s_1) - k_1$$

$$v(s_2) = w(s_2) + k_2$$

$$v_s(s_1) = w_s(s_1)$$

$$v_s(s_2) = w_s(s_2)$$

Figure 2 below displays the output screen of the model implementation applied to a 250 acre cropland in Illinois.

Insert Figure 2 here

The screen design layout consists of a separation of the input and output parameters. Once the data parameters have been input and initialised, the algorithm calculates the cropland value in addition to the critical spot prices that determines the optimal management operational policies s_0 (abandon operations), s_1 (cease operations) and s_2 (commence operations) - which are $2.59, $2.78 and $3.90 respectively.

The specific cropland values and their relative risk-adjusted discount rates are shown when a particular spot price is entered into the operation located at the bottom left of screen. The value of an asset currently being operated is displayed as v(s), and an asset that is currently untapped as w(s) in millions of dollars to the nearest ten thousand dollars. For instance, Figure 2 above shows that for a corn price of $5.00, and following the appropriate managerial policies outlined above (the asset is assumed to be operational since this price is above all critical spot prices), the 250-acre cropland in Illinois is valued at $780,000 which implies a 7.01% risk-adjusted discount rate for that particular value.

A graphical representation of the two separate output functions for both operating states v(s) (operational) and w(s) (mothballed) are displayed at the top right-hand side of the screen. The phase space is defined by the cropland value in millions of dollars on the vertical axis and the current corn spot price in dollars per bushel on the horizontal axis providing the full range of cropland values for corn prices in the range of $0.00 - $10.00 per bushel. In keeping with the specifications of the model, by which it is assumed the managerial policies are being followed appropriately in order to optimise the value of the asset, only the portion of the v(s) function (displayed in black) above the critical price S1, and the portion of the function w(s) (displayed in red) below the critical price S2 are shown.

For this specific application the black section displays the value of the cropland asset v(s) when being operated above the critical spot price \$2.78. It is optimal to maintain operations of the cropland until the spot price falls below this price where the value of the operational cropland is worth less than the value of the untapped (mothballed) cropland plus the \$10,000 cost to close down operations. The red section shows the value of a currently untapped asset below a corn price of \$3.90. It is optimal to begin operating an untapped asset above this price because the value of the operating asset is worth more than to the value of the untapped asset plus the \$10,000 cost to commence operations.

Table 4 below presents the results for the calculations of cropland value as a function of the corn price for the various States. The table also presents values of the associated risk-adjusted discount rates that would be applicable in a corresponding DCF analysis which would produce identical results for this particular range of spot prices. These discount rates are calculated using the novel incorporation of the real options approach into the traditional present value framework for perpetuities.

Insert Table 4

The results contained in Table 4 show that cropland values per planted acre vary significantly according to location in the U.S. while the size of the cropland has a rather smaller impact. It can be seen that cropland located in the States of Iowa and Illinois have the highest values per-acre at each price interval, while values in Indiana and Ohio constitute an apparent mid-range across all states, and values calculated for Missouri are significantly lower. This is unsurprising given that farms in Iowa and Illinois have relatively low production costs while Missouri farms have higher costs.

Note that although the hypothetical spot price \$3.50 is below the actual average costs of production for several states (i.e. on average farmers in these states are currently out-of-the-money) the option values of the cropland remain positive at this price. The observable negative discount rates associated with this price level are the result of equating the DCF valuation with the real options valuation. Given a positive options value when the free cash flows from the DCF model are negative this must mathematically result in a corresponding negative discount rate.

It is of interest to compare these model results with the official estimates of average cropland value published by the USDA for each State. The USDA estimates are calculated using a complete, probability-based land-area sampling frame based on annual survey data which includes a stratified sample of land areas averaging approximately one square mile in size gathered during the first two weeks of June (USDA, 2011e). Table 5 below shows the most recent cropland values reported by the USDA for the years 2006 to 2010.

Insert Table 5

Unfortunately, the USDA does not disaggregate their data to distinguish between specific uses of cropland (for example, corn, sorghum, or wheat) within each of the States. Nevertheless, it is noted that a similar pattern can be observed between the USDA values and the model values, in that Illinois and Iowa have the highest cropland values, while Missouri has significantly lower values. This raises an interesting question: what is the appropriate spot price at which the values obtained by implementing the renewable resource model match the official USDA estimates? In Table 6 we report the appropriate spot price at which the model produces cropland values equivalent to those reported by the USDA for 2010.

Insert Table 6

It can be seen in the table above that a spot price in the range \$5.45 to \$6.12 would match the official cropland values published by the USDA for 2010. To see how these calculations compare with observed monthly corn prices received by U.S. farmers a time-series of these prices was produced using monthly prices reported by the USDA for January 2010 to June 2011. This time-series is presented below in Figure 3.

Insert Figure 3

Figure 3 shows that during 2010 corn prices were significantly below the calculations of the appropriate prices at which the model matches the USDA values. However, prices observed in 2011 from April to June are found to be higher than these "benchmark" prices.

These results can be interpreted in at least several ways. First, they suggest that, if the renewable resource model's assumptions hold and if the model parameters adopted above are a valid representation of the true situation, then the market valuations of corn cropland in the states studied may have involved some over-valuation during 2010, in that the corn prices which prevailed during the period implied (according to the model) lower cropland values. By the same token, the market valuations in much of 2011 appear to have been somewhat lower than indicated by the model.

Second, the above characterisation of over- and under-valuation needs to be qualified by the fact that USDA valuations of cropland related to the average for all crops, whereas the model-generated valuations are geared to corn croplands only. It is possible, for example, that during 2010 the returns on crops other than corn may have been, and were expected to remain, higher than returns on corn, so that market values of all croplands combined would be higher than those for corn croplands. In that case, market values of corn croplands may have been far closer to the model-based values than the above comparison may suggest.

Third, the discussion in the above two paragraphs notwithstanding, it is of interest to note how close the model's calculations turn out to be relative to market valuations, as indicated by USDA estimations. Recall that in parameterising the model and implementing the solution methods in this case study, we have had to make a number of simplifying assumptions and work with “ballpark” estimation, such as those for production costs. Despite these approximations, the results turn out to be certainly quite close to USDA estimates. This suggests that, in a real-world situation, it may well be worth investing additional resources into refining the estimates of the relevant model parameters in order to obtain more accurate model valuations. In particular, compiling cost and tax data that is more specific for individual farms would provide a more robust empirical analysis, and would seem a worthwhile avenue for future research.

Under the model, it is optimal to continue perennial corn farming operations on the crop land until the price falls to the critical price S1. If the price falls below this level, it is optimal to delay planting for the coming season and instead incur the costs of closure and maintenance of the land to minimise operational losses which would have resulted from going ahead with planting. It is optimal to abandon production of corn entirely when the price falls to S0 where the option value to operate a corn crop on this land is zero. Conversely, if the crop land is currently not operational (that is the spot price has been low enough to warrant delaying planting this seasons corn crop), and the price rises above the critical price level S2, it is optimal to return to normal operating conditions and incur the associated production costs. The results of the calculations for these critical prices is presented below in Table 7 below.

Insert Table 7

Illinois and Iowa are found to have lower price thresholds at which it becomes optimal to exercise the various operational options over a farm. This can be expected given that our calculations showed farmers in these states to have the lowest costs of production in the Heartland Region. It can be seen that the larger the farm size, the lower the critical price, S1, at which it becomes optimal to abandon corn-farming operations. Similarly, a larger farm size reduces the critical price to reinstate operations, S2, over a previously mothballed crop. A larger farm size, on the other hand, increases the price at which it becomes optimal to temporarily delay corn farming. In other words, the larger the corn crop size, the smaller the price range between critical prices S1 and S2, and the larger the price range between critical prices S0 and S1.

There were no instances where the corn price fell below the highest calculated abandonment price of \$3.14 (for both small and large farms) for the monthly, USDA data for average U.S., corn prices received by farmers over the period January 2009 to May 2011. In fact the last time the monthly price was below this level was in January 2007. The monthly corn price fell below the critical price to cease production, S1, for both small and large farms in Missouri in 2009 during the months of August and September. Similarly in 2010 the price fell below these prices for large farms on consecutive months from February to July.

Farmers can stockpile inventory for future delivery when market conditions improve. Furthermore, for many farms, the decision to continue unprofitable operations may be influenced by both contractual obligations with various end-users of corn and with efforts to ensure marketing networks are maintained. Investigating how these relatively low prices affected farming operations amongst individual farms in Missouri (as the model suggests was appropriate) and, more generally, looking for evidence of hysteresis inertia in output level affecting operational decision-making in agriculture, would be an interesting avenue for further research.

Durst (2009) notes that nearly three-quarters of farm sole proprietors in the U.S. reported a farm loss in 2006 and that for small residential farms in particular the return to farming from the tax code may partially explain continued farm production despite the persistence of farm losses. Losses from farming are used to reduce taxes on other income which is especially the case for individuals who report their primary occupation to be something other than farming. Similarly, large commercial farms may not be stand-alone businesses, but rather part of a broader portfolio of assets for which losses may be offset against other parts of the firm (see Moel & Tufano, 2002). This implies that for large, well-diversified firms, operating policy decisions may be made at the firm level rather than at the operational level. Other non-financial considerations may also influence the investment decision process. For instance, Tauer (2006) posited that farmers often hold optimistic expectations of the future in that next month's price might be better than that at present and thus continue operations despite an objective financial analysis indicating that it would be appropriate to do otherwise.

5. Conclusion

The Renewable Resource Model extends the Brennan and Schwartz (1985) framework by recasting that framework to suit assets that have a renewable inventory under the assumption that the rate of extraction does not exceed the rate at which the resource is replenished. An integral part of the derivation of the new model is the specification of boundary conditions similar to, albeit mathematically distinct from, the boundary conditions of the original framework. We show the resultant analytically continued model's equations can be solved to produce closed form solutions. Specific solutions were then calculated via numerical techniques and consequently computational implementations of general solutions for the final model were developed.

The Renewable Resource Model valuations were consistent with official estimates of cropland values published by the USDA, although there were differences which can be explained in terms of fairly plausible factors, and which should be further investigated in the future. For example, crop values in the States of Illinois and Iowa had the highest cropland values and Missouri values were significantly lower. Our results further indicate significant differences between optimal operational decision-making for relatively small and large corn farms.

For larger farms the trigger prices at which it becomes appropriate to recommence operations on farms in which operations had been temporarily mothballed were relatively lower than for smaller sized farms, while the appropriate crop abandonment prices were also lower for large farms. Prices at which it is appropriate to delay production to avoid operating losses, however, were higher for larger farms. As one may expect, seasonal entry and exit prices as well as crop abandonment prices were found to be lower for relatively low-cost producers than high-cost producers

The location and relative size of corn croplands were also found to be important factors affecting the critical prices at which to exercise various operational options in order to manage operations optimally. Illinois and Iowa, which are relatively low-cost producers, were found to have lower price thresholds at which it becomes optimal to exercise all three options. Across all states, larger croplands were found to have lower critical prices at which it becomes optimal to abandon corn-farming operations than smaller croplands. The differences between 250- and 1000-acre croplands ranged from 4 to 6 cents.

Larger croplands were also found to have higher prices at which it becomes optimal to temporarily cease corn farming in all cases. The relatively larger price ranges between the critical prices to shut down and abandon show that it is optimal to exercise the option to cease operating larger croplands at a higher price but abandon at a lower price than for small croplands. Conversely, smaller croplands had higher critical prices to exercise the option to commence operations. Although small farms require higher price thresholds at which it becomes optimal to commence operating their croplands, should the price fall once operations begin, it is optimal to sustain operations at lower price thresholds before exercising the option to cease operations.

The model provides a basis for evaluating agricultural enterprises taking into account price volatility and managerial flexibility. Our model also provides an objective approach to determining the appropriate operating policy for these farmers; however, their decisions may ultimately be affected by considerations that include subjective factors outside the scope of the model. This valuation model will become of increasing importance as competition between nations for world food resources increases. The key contribution of the current research is to provide farmers with a means to inform their overall decisions using a purely objective financial analysis.

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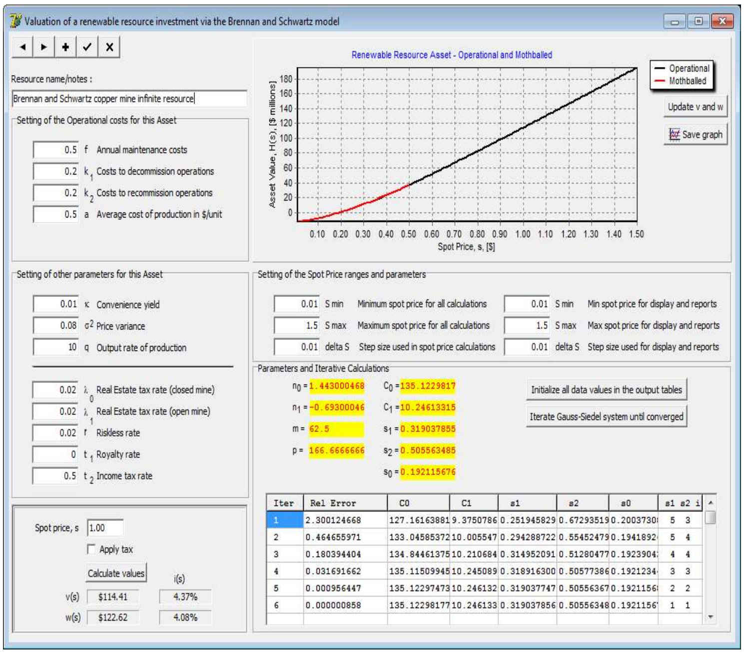
Figure 1 – The Solution Implementation Screen for a Renewable Resource Asset



Iter	Rel Error	C0	C1	s1	s2	s0	s1	s2
1	2.300124668	127.16163881	9.3750786	0.251945829	0.67293519	0.2003730	5	3
2	0.464655971	133.04585372	10.005547	0.294288722	0.55452479	0.1941892	5	4
3	0.180394404	134.84461375	10.210684	0.314952091	0.51280477	0.1923904	4	4
4	0.031691662	135.11509945	10.245089	0.318916300	0.50577386	0.1921234	3	3
5	0.000956447	135.12297473	10.246132	0.319037747	0.50556367	0.1921156	2	2
6	0.000000858	135.12298177	10.246133	0.319037856	0.50556348	0.1921156	1	1

Figure 2: Evaluation of Average Corn Crop in Illinois, 250 Acres

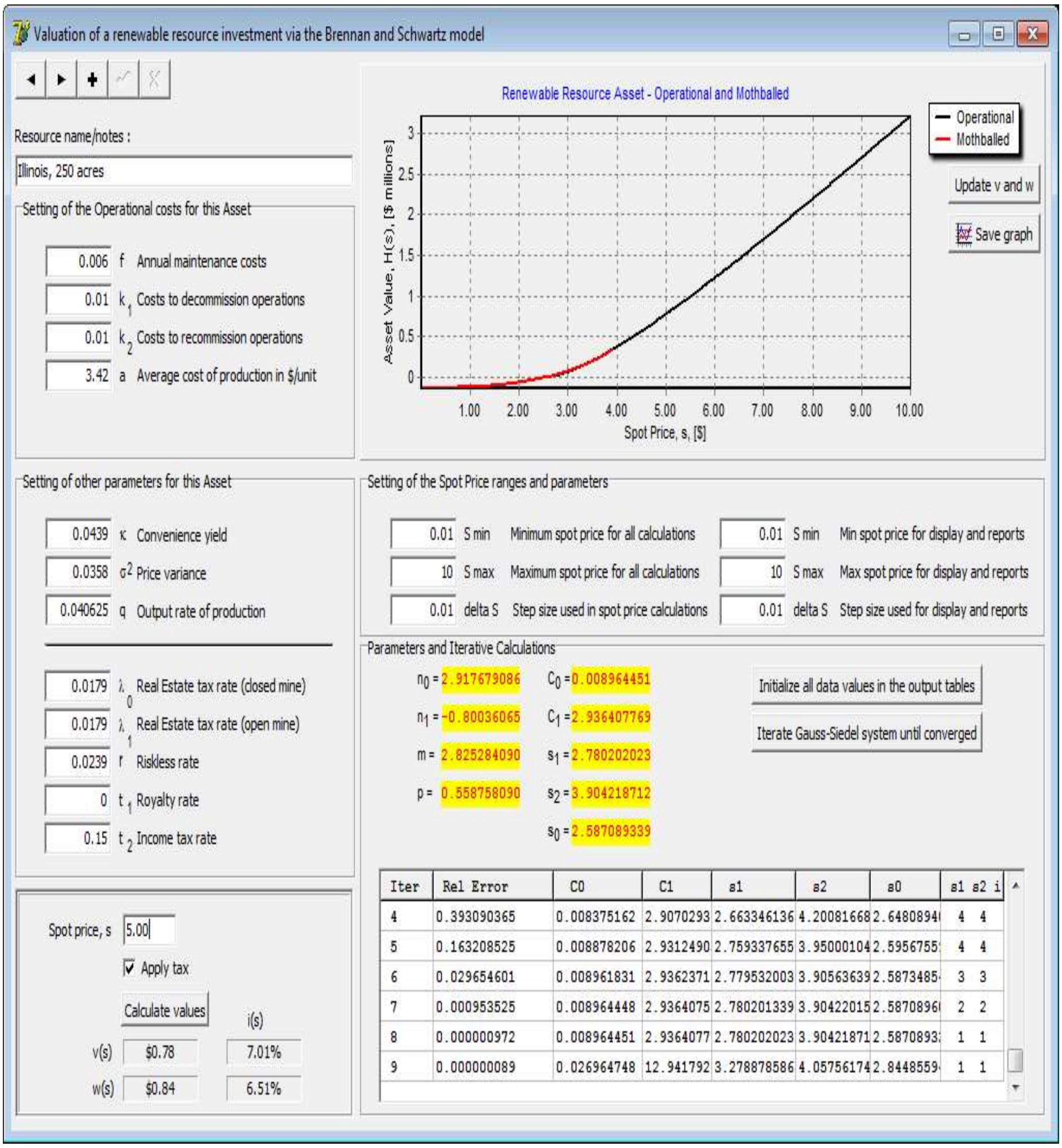
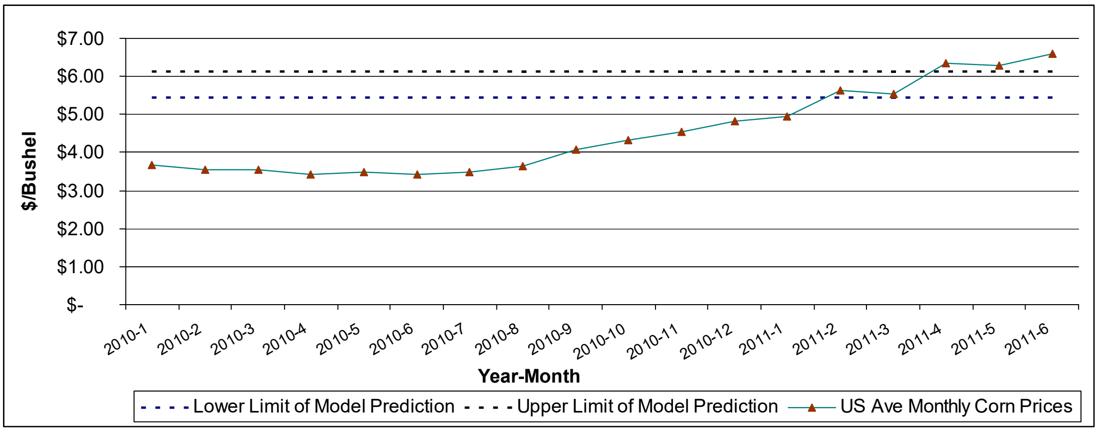


Figure 3: U.S. Average Monthly Corn Prices



Source: USDA, 2011b.

Table 1: US Corn Belt Average Annual Harvest Yields

	Average Yield by State* (bu/planted acre)				
Year	Illinois	Indiana	Iowa	Missouri	Ohio
2000	151	147	145	143	147
2001	150	160	147	136	144
2002	136	121	165	105	88
2003	169	150	159	109	156
2004	180	168	181	162	158
2005	145	151	175	108	143
2006	165	159	163	142	161
2007	175	155	171	142	150
2008	179	160	172	140	140
2009	175	166	183	151	166
Average	162.5	153.7	166.1	133.8	145.3
Production Cost** ($/bu)	$3.42	$3.61	$3.34	$4.15	$3.82

* *United States Department of Agriculture (2010d)*

** *Assuming total costs of $555.15 per planted acre (USDA, 2010c), Average cost = average total economic costs per planted acre / average yield per planted acre*

Table 2: Calculation for Average Convenience Yield (Corn) 2000-2009.

Year-Month	Average US Corn Price* $/bushel	$\ln(S_1/S_0)$	10yr US Bond Rate** % per year	Convenience Yield*** % (annualised)	Year-Month	Average US Corn Price* $/bushel	$\ln(S_1/S_0)$	10yr US Bond Rate** % per year	Convenience Yield*** % (annualised)
1999-12	1.82				**2005-01**	2.12	0.0385	4.22	3.76
2000-01	1.91	0.0483	6.66	6.08	**2005-02**	1.95	-0.0836	4.17	5.17
2000-02	1.98	0.0360	6.52	6.09	**2005-03**	2.02	0.0353	4.5	4.08
2000-03	2.03	0.0249	6.26	5.96	**2005-04**	2	-0.0100	4.34	4.46
2000-04	2.03	0.0000	5.99	5.99	**2005-05**	1.98	-0.0101	4.14	4.26
2000-05	2.11	0.0387	6.44	5.98	**2005-06**	2.03	0.0249	4	3.70
2000-06	1.91	-0.0996	6.1	7.30	**2005-07**	2.11	0.0387	4.18	3.72
2000-07	1.64	-0.1524	6.05	7.88	**2005-08**	1.95	-0.0789	4.26	5.21
2000-08	1.52	-0.0760	5.83	6.74	**2005-09**	1.9	-0.0260	4.2	4.51
2000-09	1.61	0.0575	5.8	5.11	**2005-10**	1.82	-0.0430	4.46	4.98
2000-10	1.74	0.0777	5.74	4.81	**2005-11**	1.77	-0.0279	4.54	4.87
2000-11	1.86	0.0667	5.72	4.92	**2005-12**	1.92	0.0813	4.47	3.49
2000-12	1.97	0.0575	5.24	4.55	**2006-01**	2	0.0408	4.42	3.93
2001-01	1.98	0.0051	5.16	5.10	**2006-02**	2.02	0.0100	4.57	4.45
2001-02	1.96	-0.0102	5.1	5.22	**2006-03**	2.06	0.0196	4.72	4.48
2001-03	1.96	0.0000	4.89	4.89	**2006-04**	2.11	0.0240	4.99	4.70
2001-04	1.89	-0.0364	5.14	5.58	**2006-05**	2.17	0.0280	5.11	4.77
2001-05	1.82	-0.0377	5.39	5.84	**2006-06**	2.14	-0.0139	5.11	5.28
2001-06	1.76	-0.0335	5.28	5.68	**2006-07**	2.14	0.0000	5.09	5.09
2001-07	1.87	0.0606	5.24	4.51	**2006-08**	2.09	-0.0236	4.88	5.16
2001-08	1.9	0.0159	4.97	4.78	**2006-09**	2.2	0.0513	4.72	4.10
2001-09	1.91	0.0052	4.73	4.67	**2006-10**	2.55	0.1476	4.73	2.96
2001-10	1.84	-0.0373	4.57	5.02	**2006-11**	2.88	0.1217	4.6	3.14
2001-11	1.85	0.0054	4.65	4.58	**2006-12**	3.01	0.0441	4.56	4.03
2001-12	1.98	0.0679	5.09	4.28	**2007-01**	3.05	0.0132	4.76	4.60
2002-01	1.97	-0.0051	5.04	5.10	**2007-02**	3.44	0.1203	4.72	3.28
2002-02	1.93	-0.0205	4.91	5.16	**2007-03**	3.43	-0.0029	4.56	4.59
2002-03	1.94	0.0052	5.28	5.22	**2007-04**	3.39	-0.0117	4.69	4.83
2002-04	1.91	-0.0156	5.21	5.40	**2007-05**	3.49	0.0291	4.75	4.40
2002-05	1.93	0.0104	5.16	5.03	**2007-06**	3.53	0.0114	5.1	4.96
2002-06	1.97	0.0205	4.93	4.68	**2007-07**	3.32	-0.0613	5	5.74
2002-07	2.13	0.0781	4.65	3.71	**2007-08**	3.26	-0.0182	4.67	4.89
2002-08	2.38	0.1110	4.26	2.93	**2007-09**	3.28	0.0061	4.52	4.45
2002-09	2.47	0.0371	3.87	3.42	**2007-10**	3.29	0.0030	4.53	4.49
2002-10	2.34	-0.0541	3.94	4.59	**2007-11**	3.44	0.0446	4.15	3.61
2002-11	2.28	-0.0260	4.05	4.36	**2007-12**	3.77	0.0916	4.1	3.00
2002-12	2.32	0.0174	4.03	3.82	**2008-01**	3.98	0.0542	3.74	3.09
2003-01	2.33	0.0043	4.05	4.00	**2008-02**	4.54	0.1316	3.74	2.16
2003-02	2.34	0.0043	3.9	3.85	**2008-03**	4.7	0.0346	3.51	3.09
2003-03	2.33	-0.0043	3.81	3.86	**2008-04**	5.14	0.0895	3.68	2.61
2003-04	2.34	0.0043	3.96	3.91	**2008-05**	5.27	0.0250	3.88	3.58
2003-05	2.38	0.0169	3.57	3.37	**2008-06**	5.47	0.0372	4.1	3.65
2003-06	2.34	-0.0169	3.33	3.53	**2008-07**	5.25	-0.0411	4.01	4.50
2003-07	2.17	-0.0754	3.98	4.89	**2008-08**	5.26	0.0019	3.89	3.87
2003-08	2.15	-0.0093	4.45	4.56	**2008-09**	5.01	-0.0487	3.69	4.27
2003-09	2.2	0.0230	4.27	3.99	**2008-10**	4.37	-0.1367	3.81	5.45
2003-10	2.12	-0.0370	4.29	4.73	**2008-11**	4.26	-0.0255	3.53	3.84
2003-11	2.2	0.0370	4.3	3.86	**2008-12**	4.11	-0.0358	2.42	2.85
2003-12	2.31	0.0488	4.27	3.68	**2009-01**	4.36	0.0590	2.52	1.81
2004-01	2.39	0.0340	4.15	3.74	**2009-02**	3.87	-0.1192	2.87	4.30
2004-02	2.61	0.0881	4.08	3.02	**2009-03**	3.85	-0.0052	2.82	2.88
2004-03	2.75	0.0523	3.83	3.20	**2009-04**	3.85	0.0000	2.93	2.93
2004-04	2.89	0.0497	4.35	3.75	**2009-05**	3.96	0.0282	3.29	2.95
2004-05	2.87	-0.0069	4.72	4.80	**2009-06**	4.01	0.0125	3.72	3.57
2004-06	2.79	-0.0283	4.73	5.07	**2009-07**	3.6	-0.1079	3.56	4.85
2004-07	2.51	-0.1058	4.5	5.77	**2009-08**	3.33	-0.0780	3.59	4.53
2004-08	2.34	-0.0701	4.28	5.12	**2009-09**	3.25	-0.0243	3.4	3.69
2004-09	2.2	-0.0617	4.13	4.87	**2009-10**	3.61	0.1051	3.39	2.13
2004-10	2.14	-0.0277	4.1	4.43	**2009-11**	3.65	0.0110	3.4	3.27
2004-11	2.05	-0.0430	4.19	4.71	**2009-12**	3.6	-0.0138	3.59	3.76
2004-12	2.04	-0.0049	4.23	4.29	**Average**	**\$2.68**	**0.0057**	**4.46%**	**4.39%**

* *United States Department of Agriculture (2011b)*

** *Board of Governors of the Federal Reserve System (2011)*

*** *Calculated as cy=r-12*ln(s1/s0), see Gibson & Schwartz (1990)*

Table 3: Data Parameters Summary

Parameter		Value
Real Interest Rate		2.39% per year
Annual Volatility*		18.91% per year
Average Convenience Yield		4.39% per year
Average Annual Yield		
	Illinois	162.5 bu/planted acre
	Indiana	153.7 bu/planted acre
	Iowa	166.1 bu/planted acre
	Missouri	133.8 bu/planted acre
	Ohio	145.3 bu/planted acre
Average Production Cost		
	Illinois	$3.42 per bushel
	Indiana	$3.61 per bushel
	Iowa	$3.24 per bushel
	Missouri	$4.15 per bushel
	Ohio	$3.82 per bushel
Maintenance Cost		$24 per planted acre
Cost to Temporarily Close		$10,000
Cost to Reinstate Operations		$10,000
Federal Income Tax Rate		15%
State Propoerty Tax Rate		
	Illinois	1.79% per year
	Indiana	2.12% per year
	Iowa	2.15% per year
	Missouri	1.42% per year
	Ohio	1.81% per year
Royalty Rate		0%

** Variance = 0.0358*

Table 4: Real Options Evaluation of Corn Cropland and Risk-adjusted Discount Rates at Various Spot Prices

250 acres	Illinois		Indiana		Iowa		Missouri		Ohio	
Spot Price ($/bu)	Crop Value ($/acre)	i(s) (%)	Crop Value ($/acre)	i(s) (%)	Crop Value ($/acre)	i(s) (%)	Crop Value ($/acre)	i(s) (%)	Crop Value ($/acre)	i(s) (%)
$3.50*	840	1.33	560	-2.52	960	3.77	200	-37.90	400	-9.69
$4.00**	1520	5.30	1160	4.45	1680	6.40	640	-2.70	920	2.39
$4.50	2280	6.54	1800	6.43	2440	7.24	1160	3.39	1560	5.43
$5.00	3120	7.01	2520	7.17	3280	7.54	1800	5.38	2240	6.53
$5.50	4000	7.20	3320	7.47	4160	7.65	2480	6.21	2960	6.99
$6.00	4920	7.26	4120	7.58	5080	7.67	3200	6.58	3760	7.19
1000 acres	**Illinois**		**Indiana**		**Iowa**		**Missouri**		**Ohio**	
Spot Price ($/bu)	Crop Value ($/acre)	i(s) (%)	Crop Value ($/acre)	i(s) (%)	Crop Value ($/acre)	i(s) (%)	Crop Value ($/acre)	i(s) (%)	Crop Value ($/acre)	i(s) (%)
$3.50*	890	1.24	630	-2.29	1020	3.58	260	-28.14	470	-8.42
$4.00**	1560	5.13	1200	4.26	1720	6.23	690	-2.46	980	2.26
$4.50	2330	6.41	1850	6.27	2500	7.12	1230	3.23	1600	5.26
$5.00	3160	6.92	2580	7.05	3330	7.46	1850	5.23	2280	6.39
$5.50	4030	7.12	3350	7.38	4210	7.58	2520	6.09	3010	6.89
$6.00	4950	7.21	4150	7.52	5110	7.62	3240	6.49	3790	7.11

** Spot price is below average cost of production for Indiana, Missouri and Ohio*

*** Spot price is below average production cost for Missouri*

Table 5: USDA Average Cropland Values by State

Year	Illinois	Indiana	Iowa	Missouri	Ohio
2006	$ 3,640	$ 3,250	$ 3,100	$ 2,010	$ 3,470
2007	$ 4,150	$ 3,640	$ 3,600	$ 2,330	$ 3,820
2008	$ 4,850	$ 4,140	$ 4,260	$ 2,500	$ 4,140
2009	$ 4,670	$ 3,950	$ 4,050	$ 2,540	$ 3,900
2010	$ 4,820	$ 4,030	$ 4,100	$ 2,670	$ 3,950

Source: USDA, 2011e

Table 6: Results for the Appropriate Corn Prices at which the Model matches the USDA estimates for Cropland Values

	Spot Prices ($/bu)				
Crop Size	Illinois	Indiana	Iowa	Missouri	Ohio
250 acres	$5.95	$5.95	$5.45	$5.64	$6.12
1000 acres	$5.93	$5.93	$5.44	$5.60	$6.10

Table 7: Real Options Analysis of Critical Option Prices

250 acres	S0*	S1**	S2***
Illinois	$2.59	$2.78	$3.90
Indiana	$2.74	$2.93	$4.12
Iowa	$2.48	$2.62	$3.70
Missouri	$3.14	$3.37	$4.74
Ihio	$2.89	$3.10	$4.36
1000 acres	**S0**	**S1**	**S2**
Illinois	$2.55	$2.94	$3.63
Indiana	$2.69	$3.10	$3.83
Iowa	$2.44	$2.77	$3.44
Missouri	$3.08	$3.56	$4.41
Ihio	$2.84	$3.28	$4.06

** S0 - abandon*

*** S1 - temporarily cease production*

**** S2 - recommence production*