Multivariate decompositions with correlated innovations*

Jing Tian†
University of Tasmania and CAMA

Jan P.A.M. Jacobs
University of Groningen, CAMA and CIRANO

Denise R. Osborn
University of Manchester and CAMA

December 2021

Abstract

Multivariate analysis can help to focus on economic phenomena, including trend and cyclical movements, but seasonality is often ignored. The present paper studies a multivariate unobserved component model for quarterly data with a general correlation structure across trend, cycle and seasonal components, showing that economic restrictions, including common trends, and common cycles, can ensure identification. An application to seasonal aggregate gender employment in Australia leads to a common cycle specification for male and female employment.

JEL classification: C32; E24; E32
Keywords: trend-cycle-seasonal decomposition; multivariate unobserved components models; correlated component models; identification; gender employment; Australia

*This paper is dedicated to Mardi H. Dungey (1966-2019), our friend, co-author and mentor. A preliminary version was presented at the Mardi Dungey Memorial Research Conference, Washington DC, 21 February 2020, organised by Renee Fry-McKibbin and Tara Sinclair. We are also grateful to James Morley, Simon van Norden and especially Tara Sinclair for insightful comments on an earlier version of this paper.

†Correspondence to Jing Tian, Tasmanian School of Business and Economics, Private Bag 84, University of Tasmania, Tasmania, 7001, Australia, Jing.Tian@utas.edu.au

1 Introduction

Over many decades, economists have sought to understand the drivers underlying the time series evolution of series of interest through the analysis of seasonally adjusted data. The notion that a series can be meaningfully decomposed into components that are not directly observed is, therefore, deeply embedded in empirical economic analyses.

An important strand of literature employs so-called unobserved components (UC) models to study trends and cycles as separate phenomena of economic interest. While conventional UC models assume that the individual components are uncorrelated, a substantial literature questions this in the context of analyzing trend and cyclical movements in seasonally adjusted data; important contributions include Clark (1989), Morley, Nelson and Zivot (2003, henceforth MNZ), Morley (2007), Sinclair (2009), Dungey et al. (2013, 2015). However, the potential consequences of allowing seasonality to be correlated with other components are much more pervasive because the uncorrelated assumption is implicit in the widespread use of seasonally adjusted data. Indeed, Wright (2013) argues that conventional seasonal adjustment has distorted the true effects of the Great Recession, while Stock (2013) points to the difficulties of determining an appropriate seasonal adjustment filter. From a different perspective, Cecchetti and Kashyap (1996), Krane and Wascher (1999) and Matas-Mir and Osborn (2004), among others, argue on both economic and statistical grounds that cyclical and seasonal components may be correlated.

Our approach follows MNZ and others by using a UC framework that takes account of interactions through correlated innovations, but we extend the framework to explicitly model seasonality. Further, since multivariate analysis can throw important light on underlying economic phenomena, such as common trends and common cycles (for example, Morley (2007), Fleischman and Roberts (2011)), we employ a three component multivariate UC model. Very recently, Hindrayanto, Jacobs, Osborn and Tian (2019, henceforth HJOT) consider such a trend-cycle-seasonal model in the univariate context and show the imposition of a single correlation restriction is sufficient for identification. Building on HJOT, the present paper focuses on a multivariate model for quarterly data, examining identification and how this can be achieved through the use of economically plausible restrictions, such as common trends or common cycles. To our knowledge, no previous analysis has examined identification conditions for a correlated multivariate UC trend-cycle-seasonal model.

To illustrate the range of models that can result in practice, the framework is applied to seasonal gender employment in Australia. More specifically, a bivariate male/female model with a common cycle is preferred to other univariate and bivariate specifications considered.

The paper proceeds as follows. Section 2 discusses multivariate correlated seasonal UC models. It is shown that while the general model is not identified, plausible economic restrictions can allow identification in the presence of non-zero correlations between trend, cycle and seasonal shocks. Section 3 then illustrates the approach through an application to gender employment in Australia. The final section concludes.

2 Multivariate UC Models

This section describes the model and discusses its identification, including economically plausible restrictions that may apply.

2.1 Seasonal UC model

Many macroeconomic variables exhibit trend, cycle and seasonal characteristics. Hence, for an observed $k \times 1$ vector $\boldsymbol{Y}_t$, consider a multivariate UC model that explicitly recognizes these characteristics through the measurement equation

$$\boldsymbol{Y}_t = \boldsymbol{T}_t + \boldsymbol{C}_t + \boldsymbol{S}_t, \tag{1}$$

in which the trend, cycle and seasonal components ($\boldsymbol{T}_t$, $\boldsymbol{C}_t$ and $\boldsymbol{S}_t$, respectively) are, in general, each $k \times 1$ vectors.

Following MNZ and many others, we assume that the trend for each variable is $I(1)$ and can be represented as a random walk with drift, so that

$$\boldsymbol{T}_t = \boldsymbol{T}_{t-1} + \boldsymbol{\beta} + \boldsymbol{\eta}_t, \tag{2}$$

where $\boldsymbol{\eta}_t = (\eta_{1t}, \ldots, \eta_{kt})'$, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_k)$ and the $k \times k$ covariance matrix $E[\boldsymbol{\eta}_t\boldsymbol{\eta}_t'] = \boldsymbol{\Sigma}_{\boldsymbol{\eta\eta}}$ is not *a priori* assumed to be diagonal. The multivariate cyclical component of (1) is represented by the AR processes

$$\boldsymbol{\Phi}(L)\boldsymbol{C}_t = \boldsymbol{\varepsilon}_t, \tag{3}$$

where $\boldsymbol{\Phi}(L)$ is a $k \times k$ matrix in the lag operator L, with $\boldsymbol{\Phi}(L) = \boldsymbol{I}_k - \boldsymbol{\Phi}_1 L - \ldots - \boldsymbol{\Phi}_p L^P$ ($\boldsymbol{I}_k$ being a $k \times k$ identity matrix) having all roots strictly outside the unit circle and, with $\boldsymbol{\varepsilon}_t$ defined in the obvious way, $E[\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'] = \boldsymbol{\Sigma}_{\varepsilon\varepsilon}$. As usual in economic applications of multivariate UC models, such as Morley (2007), Sinclair (2009) or Ma and Wohar (2013), $\boldsymbol{\Phi}(L)$ is assumed diagonal with the cycle in each variable having the same univariate order p. Empirical analyses typically employ $p = 2$, since this can both adequately capture short-term nonseasonal movements in economic data while also allowing the parameters of the correlated UC trend-cycle model to be identified; see MNZ for the univariate case and Trenkler and Weber (2016), hereafter TW, for a multivariate analysis[1].

As in HJOT and many other papers, seasonality is modeled using the so-called "dummy variable" form

$$\Psi(L)\boldsymbol{S}_t = \boldsymbol{\omega}_t, \tag{4}$$

where $\Psi(L)$ is the scalar annual summation polynomial over a year ($\Psi(L) = 1 + L + L^2 + L^3$ for quarterly data) and, in an obvious notation, $E[\boldsymbol{\omega}_t\boldsymbol{\omega}_t'] = \boldsymbol{\Sigma}_{\omega\omega}$.

[1]With seasonal data, it is also important that the cycle component is not conflated with the seasonal component. A low order, such as $p = 2$ can be important for this purpose, especially for quarterly data.

To facilitate later discussion, stack the UC model disturbances of (2)-(4) to form the $3k \times 1$ vector $\boldsymbol{U}_t$ as

$$\boldsymbol{U}_t = [\boldsymbol{\eta}_t', \boldsymbol{\varepsilon}_t', \boldsymbol{\omega}_t']', \tag{5}$$

and define the $3k \times 3k$ covariance matrix

$$E[\boldsymbol{U}_t\boldsymbol{U}_t'] = \boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{\eta\eta} & \boldsymbol{\Sigma}_{\eta\varepsilon} & \boldsymbol{\Sigma}_{\eta\omega} \\ \boldsymbol{\Sigma}_{\eta\varepsilon}' & \Sigma_{\varepsilon\varepsilon} & \boldsymbol{\Sigma}_{\varepsilon\omega} \\ \boldsymbol{\Sigma}_{\eta\omega}' & \boldsymbol{\Sigma}_{\varepsilon\omega}' & \boldsymbol{\Sigma}_{\omega\omega}, \end{bmatrix} \tag{6}$$

where, in an obvious notation,

$$E[\boldsymbol{\eta}_t\boldsymbol{\varepsilon}_t'] = \boldsymbol{\Sigma}_{\eta\varepsilon},\ E[\boldsymbol{\eta}_t\boldsymbol{\omega}_t'] = \boldsymbol{\Sigma}_{\eta\omega},\ E[\boldsymbol{\varepsilon}_t\boldsymbol{\omega}_t'] = \boldsymbol{\Sigma}_{\varepsilon\omega}. \tag{7}$$

Although the disturbances are possibly cross-correlated at t, they are assumed uncorrelated over time, so that

$$E[\boldsymbol{U}_{t_1}\boldsymbol{U}_{t_2}'] = \mathbf{0}, \qquad t_1 \neq t_2.$$

2.2 Reduced form

As a preliminary to identification, we consider the reduced form and autocovariances of the multivariate seasonal UC model for quarterly data[2]. It is straightforward to see that the system (1)-(4) implies the reduced form

$$\boldsymbol{\Phi}(L)\Delta_4\boldsymbol{Y}_t = \boldsymbol{\Phi}(1)\Psi(1)\boldsymbol{\beta} + \boldsymbol{\Phi}(L)\Psi(L)\boldsymbol{\eta}_t + \Delta_4\boldsymbol{\varepsilon}_t + \boldsymbol{\Phi}(L)\Delta_1\boldsymbol{\omega}_t, \tag{8}$$

where $\Delta_4 = 1 - L^4$ is the annual difference and Δ_1 is the usual first difference. In this general model, each element of $\boldsymbol{Y}_t$ is seasonally integrated (see, for example, Ghysels and Osborn,

[2]The expressions in this subsection can be easily generalized to monthly data, but somewhat different identification issues will arise.

2001, Chapter 4), due to the presence of a zero frequency unit root in its trend component (2) and the full set of unit roots at seasonal frequencies through the nonstationary seasonal process of (4). Hence annual differencing is required to reduce each univariate process in $\boldsymbol{Y}_t$ to stationarity, but this does not rule out cointegration across the components of $\boldsymbol{Y}_t$.

To focus on the disturbances, define from (8)

$$\begin{aligned} \boldsymbol{Z}_t &= \boldsymbol{A}(L)\boldsymbol{\eta}_t + (1-L^4)\boldsymbol{\varepsilon}_t + \boldsymbol{B}(L)\boldsymbol{\omega}_t \\ &= \boldsymbol{H}(L)\boldsymbol{U}_t, \end{aligned} \tag{9}$$

where

$$\begin{aligned} \boldsymbol{A}(L) &= (1+L+L^2+L^3)\boldsymbol{\Phi}(L) = \boldsymbol{I}_k + \boldsymbol{A}_1 L + ... + \boldsymbol{A}_{p+3}L^{p+3}, \\ \boldsymbol{B}(L) &= (1-L)\boldsymbol{\Phi}(L) = \boldsymbol{I}_k + \boldsymbol{B}_1 L + ... + \boldsymbol{B}_{p+1}L^{p+1}, \end{aligned} \tag{10}$$

while $\boldsymbol{U}_t$ is defined in (5) and $\boldsymbol{H}(L)$ is the $k \times 3k$ matrix

$$\begin{aligned} \boldsymbol{H}(L) &\equiv \begin{bmatrix} \boldsymbol{A}(L) & (1-L^4)\boldsymbol{I}_k & \boldsymbol{B}(L) \end{bmatrix} \\ &= \boldsymbol{H}_0 + \boldsymbol{H}_1 L + \boldsymbol{H}_2 L^2 + ... + \boldsymbol{H}_q H^q, \end{aligned} \tag{11}$$

where $q = \max(p+3, 4)$.

For the specific case of interest in our application, with $p = 2$ and quarterly data, then $q = 5$. Also noting that $\boldsymbol{\Phi}_1$, $\boldsymbol{\Phi}_2$ are diagonal and hence symmetric, it can easily be seen that

$$
\begin{aligned}
\boldsymbol{H}_0 &= \begin{bmatrix} \boldsymbol{I}_k & \boldsymbol{I}_k & \boldsymbol{I}_k \end{bmatrix}, \\
\boldsymbol{H}_1 &= \begin{bmatrix} \boldsymbol{A}_1 & \mathbf{0} & \boldsymbol{B}_1 \end{bmatrix} = \begin{bmatrix} (\boldsymbol{I}_k - \boldsymbol{\Phi}_1) & \mathbf{0} & -(\boldsymbol{I}_k + \boldsymbol{\Phi}_1) \end{bmatrix}, \\
\boldsymbol{H}_2 &= \begin{bmatrix} \boldsymbol{A}_2 & \mathbf{0} & \boldsymbol{B}_2 \end{bmatrix} = \begin{bmatrix} (\boldsymbol{I}_k - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2) & \mathbf{0} & (\boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2) \end{bmatrix}, \\
\boldsymbol{H}_3 &= \begin{bmatrix} \boldsymbol{A}_3 & \mathbf{0} & \boldsymbol{B}_3 \end{bmatrix} = \begin{bmatrix} (\boldsymbol{I}_k - \boldsymbol{\Phi}_1 - \boldsymbol{\Phi}_2) & \mathbf{0} & \boldsymbol{\Phi}_2 \end{bmatrix}, \\
\boldsymbol{H}_4 &= \begin{bmatrix} \boldsymbol{A}_4 & -\boldsymbol{I}_k & \mathbf{0} \end{bmatrix} = \begin{bmatrix} -(\boldsymbol{\Phi}_1 + \boldsymbol{\Phi}_2) & -\boldsymbol{I}_k & \mathbf{0} \end{bmatrix}, \\
\boldsymbol{H}_5 &= \begin{bmatrix} \boldsymbol{A}_5 & \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\Phi}_2 & \mathbf{0} & \mathbf{0} \end{bmatrix}.
\end{aligned}
$$

Since $\boldsymbol{\Phi}(L)$ is of order p and $\boldsymbol{Z}_t$ is the sum of moving averages, the reduced form (8) is a VARMA(p, q) process, with $q = p+3$ for $p > 0$. Ruling out the AR and MA polynominals in each equation $i = 1, ..., k$ having any factor in common[3], this VARMA process with diagonal $\boldsymbol{\Phi}(L)$ is identified (Dufour and Pelletier (2021, Theorem 3)). An immediate consequence is that the AR parameters in $\boldsymbol{\Phi}(L)$ are identified from the reduced form. Also, noting that $\Psi(L)$ is the (known) annual summation operator, the drift parameter vector $\boldsymbol{\beta}$ is identified through the reduced form intercept vector.

Therefore, the primary issue for identification (and discussed in the next subsection) concerns whether the elements of the covariance matrix (6) can be estimated given the values of $\boldsymbol{\Phi}(L)$ and $\boldsymbol{\beta}$. For this purpose, we consider the non-zero autocovariance matrices of $\boldsymbol{Z}_t$, namely

$$
\boldsymbol{\Gamma}_\ell = \sum_{i=0}^{q-\ell} \boldsymbol{H}_{i+\ell} \Sigma \boldsymbol{H}_i' \qquad \ell = 0, 1, ..., q. \tag{12}
$$

Using (6) and (11), (12) then yields the autocovariances of $\boldsymbol{Z}_t$ in terms of the elements of

[3]The AR polynomial $\phi_i(L)$ in the i^{th} equation will cancel in (8) when the corresponding cycle disturbance has zero variance. However, this implies the absence of a stochastic cycle component in the variable and hence $\phi_i(L)$ is not identified.

$\boldsymbol{\Sigma}$ and $\boldsymbol{\Phi}(L)$.

2.3 Covariance matrix identification

Identification proceeds by considering the relationship between the autocovariances of the moving average component of the reduced form and the covariance matrix (6) of the underlying model. MNZ show that $p \geq 2$ is sufficient for the identification of the univariate trend-cycle model, while TW generalize this result to the multivariate context. The addition of seasonality complicates identification, with HJOT showing not only that univariate models of the form (1)-(6) for quarterly data with $k = 1$ are under-identified for $p \leq 1$, but also that an additional disturbance covariance restriction is required for identification when $p = 2$.

Following the line of analysis used by the above authors, the previous subsection has already noted that $\boldsymbol{\Phi}(L)$ and $\boldsymbol{\beta}$ are identified from the multivariate ARMA reduced form. Since, from (10) and (11), the only unknowns in the matrices $\boldsymbol{H}_i$ $(i = 0, \ldots, q)$ are the AR coefficients of $\boldsymbol{\Phi}(L)$, these are also identified from the reduced form. Therefore, the autocovariances of $\boldsymbol{Z}_t$ defined by (12) can be used to provide information about the $3k(3k+1)/2$ distinct elements of $\boldsymbol{\Sigma}$, effectively treating the other parameters as given. The order condition for identification then requires $\boldsymbol{\Gamma}_\ell$ of (12) for $\ell = 0, 1, \ldots, q$ to contain at least $3k(3k + 1)/2$ distinct elements.

The $q + 1$ non-null autocovariance matrices of (12) have $qk^2 + k(k + 1)/2$ distinct elements, of which $k(k + 1)/2$ are contributed by the contemporaneous covariance matrix $\boldsymbol{\Gamma}_0$. As discussed above, the VMA order q is a consequence of both the data frequency and cycle order p. For quarterly data and $p \leq 1$, $q = 4$ and hence the number of distinct autocovariance elements in $\boldsymbol{\Gamma}_\ell$ for $\ell = 0, ..., q$, namely $(9k^2 + k)/2$, is less than the number of distinct elements of $\boldsymbol{\Sigma}$, $(9k^2 + 3k)/2$. Consequently, as for the univariate case, the parameters of the quarterly unrestricted correlated multivariate UC model with seasonality are not identified when $p \leq 1$. We therefore concentrate on the case $p = 2$, which is of interest for empirical as well as theoretical reasons.

With $p = 2$, the number of distinct elements in $\boldsymbol{\Gamma}_\ell$ of (12) for $\ell = 0, ..., 5$ is $5k^2 + k(k+1)/2$. It is easily seen that this can be written as $(11k^2 + k)/2 = (9k^2 + 3k)/2$ for $k = 1$ (the case discussed by HJOT) and $(11k^2 + k)/2 > (9k^2 + 3k)/2$ for $k > 1$. Therefore, the order condition for identification is satisfied for the correlated UC model for all values of k. However, the rank condition also needs to be satisfied and HJOT show that this fails in the univariate case.

Using a similar notation to TW, define $\boldsymbol{\gamma}_0^* = \text{vech}(\boldsymbol{\Gamma}_0)$, where the vech operator columnwise stacks the elements of $\boldsymbol{\Gamma}_0$ on and below the main diagonal into the $k(k+1)/2$ vector $\boldsymbol{\gamma}_0^*$, starting with the first column of $\boldsymbol{\Gamma}_0$ and with the elements of each subsequent column placed below the immediately preceding one. Also define the k^2 vectors $\boldsymbol{\gamma}_i = \text{vec}(\boldsymbol{\Gamma}_i)$, $i = 1, \ldots, 5$, where the conventional vec operator stacks all elements in the columns of the relevant matrix below each other. The vector $\boldsymbol{\gamma}^* = [\boldsymbol{\gamma}_0^{*\prime}, \boldsymbol{\gamma}_1', \boldsymbol{\gamma}_2', \boldsymbol{\gamma}_3', \boldsymbol{\gamma}_4', \boldsymbol{\gamma}_5']'$ then contains the $(11k^2 + k)/2$ distinct autocovariance elements for $\boldsymbol{Z}_t$ at lags $\ell = 0, \ldots, 5$. Similarly, define the vector $\boldsymbol{\sigma}^* = \text{vech}(\boldsymbol{\Sigma})$ containing the $(9k^2 + 3k)/2$ distinct elements of the component covariance matrix $\boldsymbol{\Sigma}$ and it is also possible to define a $(11k^2 + k)/2 \times (9k^2 + 3k)/2$ matrix $\boldsymbol{D}$ whose elements depend only on $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ to write the relationships as the system of equations

$$\boldsymbol{\gamma}^* = \boldsymbol{D}\boldsymbol{\sigma}^*, \tag{13}$$

in which the elements of $\boldsymbol{\sigma}^*$ are unknown. Consequently, the rank condition for identification of the unrestricted multivariate correlated UC model is that $\boldsymbol{D}$ has rank $(9k^2+3k)/2$.

HJOT show that a linear dependence exists between the autocovariances when $k = 1$ in the correlated seasonal UC model, and hence the rank condition for identification fails. A single covariance restriction on the component disturbances is then required for identification. In the multivariate case, although explicit expressions can be obtained for the elements of $\boldsymbol{D}$, these are substantially more complicated for the seasonal multivariate model than those presented by TW for the trend-cycle model or HJOT in the univariate seasonal case.

It is useful to note the dimensions of $\boldsymbol{D}$ for different values of k. In particular, for $k = 2$, $\boldsymbol{D}$ is 23×21, while for $k = 3, 4$ the dimensions are 51×45 and 90×78, respectively. Practical considerations therefore imply that the approach will be used in small systems. In the absence of an analytical solution, our proposal is to proceed by first constructing the matrix $\boldsymbol{D}$ for sets of plausible values of $\boldsymbol{\Phi}_1$ and $\boldsymbol{\Phi}_2$ (perhaps obtained from a univariate analysis), and check its rank. If the matrix is rank deficient, as HJOT show in the univariate case, then restrictions will be required for identification.

The next subsection explores sets of plausible restrictions that may be considered.

2.4 Restricted models

A conventional multivariate UC model, as used by Harvey (1989), among many others, allows the disturbances for a specific component to be correlated across variables, but imposes zero correlations across components. With trend, cycle and seasonal components, the covariance matrix of (6) is then block diagonal, with $6k^2$ zero restrictions thereby imposed on the $9k^2$ elements of $\boldsymbol{\Sigma}$. The discussion of the preceding subsection implies that this uncorrelated multivariate UC specification is over-identified.

Although some previous studies employing UC models (including Morley (2007), Ma and Wohar (2013), Clark (1989), Fleischman and Roberts (2011) and McElroy (2017)) employ restrictions across variables to improve efficiency of estimation, the inclusion of seasonality in the correlated UC model is likely to require restrictions for identification. Rather than making a *priori* assumption on the appropriate restrictions, the application of the next section considers a range of models and judges the economic plausibility of the results *ex post*.

To be specific, for $\boldsymbol{T}_t = (\tau_{1t}, \tau_{2t}, \ldots, \tau_{kt})'$, $\boldsymbol{C}_t = (c_{1t}, c_{2t}, \ldots, c_{kt})'$. $\boldsymbol{S}_t = (s_{1t}, s_{2t}, \ldots, s_{kt})'$, restrictions that may be considered include[4]:

1. Common trends, which imposes in (2)

$$\tau_{it} = d_i\tau_{1t} = d_i\tau_{1,t-1} + d_i\beta_1 + d_i\eta_{1t}, \quad i = 2, ..., k,$$

   so that both the deterministic and stochastic trend components of the ith element of T_t are the same scalar multiple d_i of τ_{1t}.

2. Common cycles, for which in (3)

$$c_{it} = b_i c_{1t}, \quad i = 2, ..., k, \tag{14}$$

   implying that $\boldsymbol{\Phi}(L) = \phi(L)\boldsymbol{B}\boldsymbol{I}_k$ where $\phi(L)$ is scalar, $\boldsymbol{B}$ is a diagonal matrix and

$$\boldsymbol{\varepsilon}_{it} = b_i\boldsymbol{\varepsilon}_{1t} \quad i = 2, ..., k. \tag{15}$$

3. Common seasonals, with $s_{it} = a_i s_{1t}$, for $i = 2, ..., k$ so that

$$\omega_{it} = a_i\omega_{1t} \quad i = 2, ..., k. \tag{16}$$

4. Perfectly correlated trend shocks, which places no restriction on the drift parameters but imposes

$$\eta_{it} = d_i\eta_{1t}, \quad i = 2, ...k. \tag{17}$$

5. Perfectly correlated cycle shocks, in which no cross-equation restrictions are placed on the AR parameters, but the cycle shocks satisfy (15).

[4]This list is not intended to be exhaustive. For example, with $k > 2$, cointegration with more than one common stochastic trend may be appropriate.

6. Same trend shock, which imposes the additional restriction $d_i = 1$ for $i = 2, ..., k$ in the perfectly correlated trend shock model.

7. Same cycle shock, which imposes the restriction $b_i = 1$ for $i = 2, ..., k$ in the perfectly correlated cycle shock model.

Of these specifications, models 1, 4 and 6 all imply the existence of a single common trend and hence $k - 1$ cointegrating relationships between the k series. The common trends model is used by Morley (2007) and Ma and Wohar (2013), with the perfectly correlated trend shock specification relaxing the implied restriction across the stochastic and deterministic trends. Clearly, the same trend shock specification restricts the stochastic trends but not the deterministic trends. The common cycle specification is used by Clark (1989), Harvey and Trimbur (2003) and Fleischman and Roberts (2011), with the perfectly correlated cycle shock specification relaxing the restriction of identical AR coefficients across variables. While the studies just mentioned consider trend and cycle components, McElroy (2017) employs a reduced rank specification of seasonality, for which common seasonality is a special case.

The illustration of the next section considers a bivariate model ($k = 2$) and Table 1 sets out the sets of restrictions considered, including the model where the only non-zero correlations are within components, namely the conventional bivariate UC model. The number of restrictions imposed by each specification is noted in the table. Although a same seasonal shock imposing $a_i = 1$ in (16) could also be considered, this was not relevant for our empirical analysis, as discussed in the next section.

Table 1: Restrictions imposed in the bivariate models estimated for male/female employment

	Uncorrelated	Common Components			Perfectly Corr. Shocks		Same Shock	
	Components	Trend	Cycle	Seasonality	Trend	Cycle	Trend	Cycle
Within component correlations		$\rho_{\eta_1\eta_2}=1$	$\rho_{\varepsilon_1\varepsilon_2}=1$	$\rho_{\omega_1\omega_2}=1$	$\rho_{\eta_1\eta_2}=1$	$\rho_{\varepsilon_1\varepsilon_2}=1$	$\rho_{\eta_1\eta_2}=1$	$\rho_{\varepsilon_1\varepsilon_2}=1$
Trend-cycle correlations	$\rho_{\eta_2\varepsilon_1}=\rho_{\eta_1\varepsilon_1}=0$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_1\varepsilon_2}=0$	$\rho_{\eta_2\varepsilon_1}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_1\varepsilon_2}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_2\varepsilon_1}$		$\rho_{\eta_2\varepsilon_1}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_1\varepsilon_2}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_2\varepsilon_1}$	$\rho_{\eta_2\varepsilon_1}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_1\varepsilon_2}=\rho_{\eta_1\varepsilon_1}$ $\rho_{\eta_2\varepsilon_2}=\rho_{\eta_2\varepsilon_1}$
Trend-seasonal correlations	$\rho_{\eta_2\omega_1}=\rho_{\eta_1\omega_1}=0$ $\rho_{\eta_2\omega_2}=\rho_{\eta_1\omega_2}=0$	$\rho_{\eta_2\omega_1}=\rho_{\eta_1\omega_1}$ $\rho_{\eta_2\omega_2}=\rho_{\eta_1\omega_2}$		$\rho_{\eta_1\omega_2}=\rho_{\eta_1\omega_1}$ $\rho_{\eta_2\omega_2}=\rho_{\eta_2\omega_1}$	$\rho_{\eta_2\omega_1}=\rho_{\eta_1\omega_1}$ $\rho_{\eta_2\omega_2}=\rho_{\eta_1\omega_2}$		$\rho_{\eta_2\omega_1}=\rho_{\eta_1\omega_1}$ $\rho_{\eta_2\omega_2}=\rho_{\eta_1\omega_2}$	
Cycle-seasonal correlations	$\rho_{\varepsilon_2\omega_1}=\rho_{\varepsilon_1\omega_1}=0$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_2}=0$		$\rho_{\varepsilon_2\omega_1}=\rho_{\varepsilon_1\omega_1}$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_2}$	$\rho_{\varepsilon_1\omega_2}=\rho_{\varepsilon_1\omega_1}$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_1}$		$\rho_{\varepsilon_2\omega_1}=\rho_{\varepsilon_1\omega_1}$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_2}$		$\rho_{\varepsilon_2\omega_1}=\rho_{\varepsilon_1\omega_1}$ $\rho_{\varepsilon_2\omega_2}=\rho_{\varepsilon_1\omega_2}$
Other restrictions		$\frac{\beta_2}{\beta_1}=\frac{\sigma_{\eta_2}}{\sigma_{\eta_1}}$	$\phi_{21}=\phi_{11}$ $\phi_{22}=\phi_{12}$				$\sigma_{\eta_2}=\sigma_{\eta_1}$	$\sigma_{\varepsilon_2}=\sigma_{\varepsilon_1}$
Number of restrictions	12	6	7	5	5	5	6	6

3 Gender employment in Australia

Birch and Preston (2020) highlight a number of gender-specific aspects of the Australian labour market, in particular noting that the female labour force participation rate reached an all-time high of 61% in 2019, only 10 percentage points below the male rate. Although this does not rule out cointegration, it suggests that the aggregate numbers of female and male employees in Australia have followed different (deterministic) trends over recent decades. Further, since more than twice as many females as males work part-time while many part-time workers in Australia are employed on a casual basis (Birch and Preston, 2020, pp.348-350), it is plausible that female employment may be more susceptible overall to cyclical and/or seasonal movements than that of males. Recent studies relating to the US (Hoynes, Miller and Schaller (2012), Guisinger (2020)) indicate that cyclical movements have gender-specific employment consequences, but do not consider seasonal aspects of employment.

Despite differences in their characteristics and responses, males and females face a common macroeconomic environment. Therefore, some commonality is anticipated across male and female employment. To exploit such commonality without imposing the essentially arbitrary assumption that seasonality is uncorrelated with other time series characteristics, we apply the bivariate correlated UC models described in the previous section to examine trend, cyclical and seasonal characteristics of male and female employment in Australia.

3.1 Preliminary analysis

Our data consists of the total number of employed persons (in thousands) by gender in Australia, provided by the Australian Bureau of Statistics[5]. We use quarterly data from 1986:Q3 to 2020:Q1, with the end-date avoiding issues arising from the Coronavirus pan-

[5]The Australian labour force data can be downloaded at `https://www.abs.gov.au/statistics/labour/employment-and-unemployment/labour-force-australia-detailed-quarterly/feb-2020#data-download`

demic. As usual, the original values are transformed by taking natural logarithms and, in order to more clearly show cyclical and seasonal movements, are multiplied by 100. Figure 1 shows that both series exhibit upward trends, with that for females steeper than for males, together with downswings during the early 1990s. Seasonality is evident in quarter-to-quarter changes, especially for females.

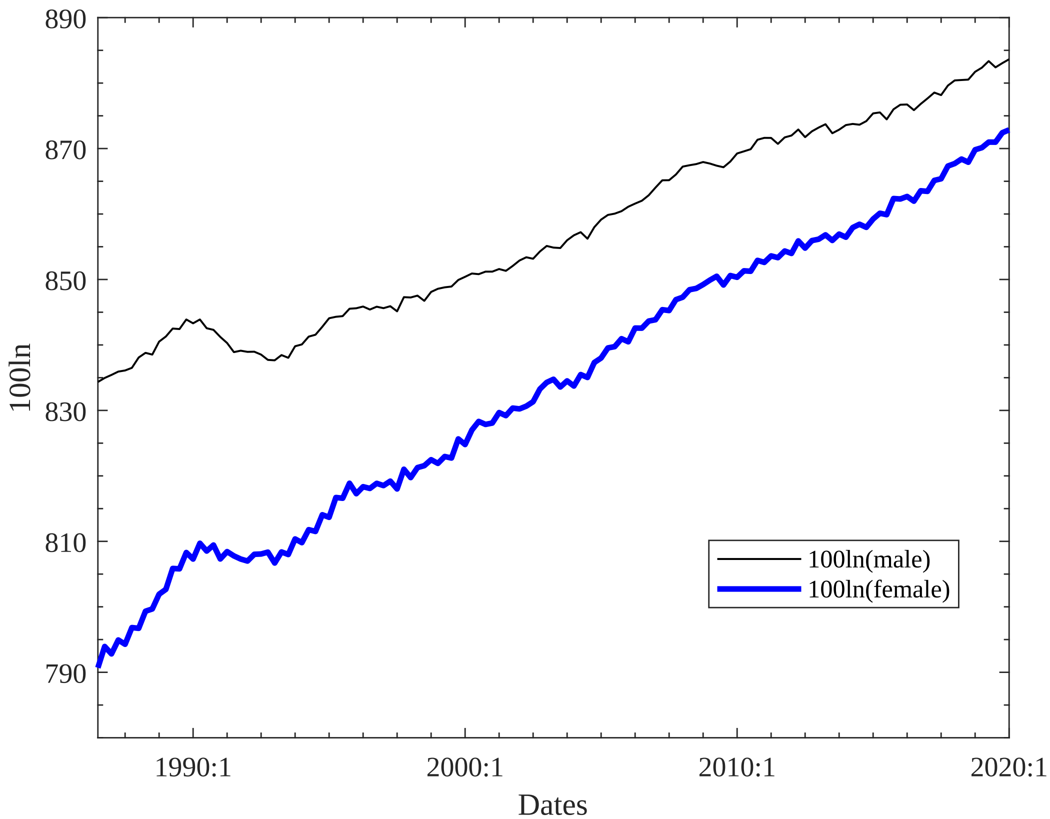

Figure 1: Male and female employment in Australia in natural logarithm times 100, 1986:Q3 to 2020:Q1

As a preliminary to our bivariate models, Table 2 reports estimation results for two univariate UC models[6]. From HJOT we know that estimation of a correlated univariate UC trend-cycle-seasonal model requires the imposition of at least one covariance restriction. The first model for each series assumes zero correlation between trend and seasonal disturbances ($\rho_{\eta\omega} = 0$). Since estimation for both series delivers estimated trend-cycle correlations ($\rho_{\eta\varepsilon}$) very close to -1, this restriction is imposed in the second model estimated for each.

Although some estimates for both series appear quite sensitive to the covariance restriction imposed, it should be borne in mind that the identification conditions for these models are only just satisfied. Identification requires the AR(2) coefficient to be non-zero and it is reassuring that the estimated values are generally significant at conventional levels, although the t-ratio in the second specification for male employment is only around 1.2. The estimated drift coefficients in Table 2 point to the steeper overall trend increase already noted for female employment compared with males.

According to AIC and BIC, the perfectly correlated trend-cycle model is preferred for male employment while the uncorrelated trend-seasonal specification is preferred for female employment. However, the estimated trend component (see Figure 2) for female employment with $\rho_{\eta\omega} = 0$ is implausible, with a 'hump' in the early 1990s. On the other hand, imposing the restriction $\rho_{\eta\varepsilon} = -1$ for female employment yields an estimated trend that closely tracks the actual data and leaves small cyclical fluctuations. The estimated components for male employment are, however, very similar from the two models. Use of a bivariate specification may assist in obtaining more satisfactory models for both series, perhaps particularly for female employment.

In order to develop a bivariate model, and as suggested in the preceding section, we first

[6] The maximum likelihood estimation results of all the UC models in this paper are obtained using MATLAB, version R2019b, with the Econometrics Toolbox$^{\text{TM}}$ state-space functionality for building the UC models in state-space forms. The elements in the covariance matrices $\mathbf{\Sigma}$ are computed via nonlinear transformation of the parameters from the state-space forms, and the delta method is used for computing the standard errors of the estimated variances and correlations for component shocks.

check the rank of the matrix $\boldsymbol{D}$ in (13) using the pairs of estimated values from the two sets univariate correlated UC models of Table 2. In both cases $\boldsymbol{D}$ has rank 19 and hence, with 21 distinct elements in $\boldsymbol{\Sigma}$, the model is under-identified and at least two restrictions need to be imposed for identification.

Table 2: Estimation results for univariate UC models for male and female employment in Australia

Parameter	Males		Females	
	$\rho_{\eta_1\omega_1} = 0$	$\rho_{\eta_1\varepsilon_1} = -1$	$\rho_{\eta_2\omega_2} = 0$	$\rho_{\eta_2\varepsilon_2} = -1$
Males:				
σ_{η_1}	1.043 (0.334)	1.068 (0.921)		
σ_{ε_1}	1.124 (0.397)	0.785 (1.072)		
σ_{ω_1}	0.019(0.010)	0.018 (0.010)		
$\rho_{\eta_1\varepsilon_1}$	-0.992 (0.014)	-1 (-)		
$\rho_{\eta_1\omega_1}$	0 (-)	-0.993 (0.043)		
$\rho_{\varepsilon_1\omega_1}$	-0.125 (0.112)	0.993 (0.043)		
Females:				
σ_{η_2}			0.823 (0.054)	0.969 (0.405)
σ_{ε_2}			0.215 (0.096)	0.402 (0.577)
σ_{ω_2}			0.040 (0.012)	0.041 (0.013)
$\rho_{\eta_2\varepsilon_2}$			-1.000 (0.000)	-1 (-)
$\rho_{\eta_2\omega_2}$			0 (-)	0.157 (0.741)
$\rho_{\varepsilon_2\omega_2}$			0.000 (0.004)	-0.157 (0.741)
Others:				
β_1	0.359 (0.083)	0.340 (0.110)		
β_2			0.619 (0.017)	0620 (0.094)
ϕ_{11}	0.588 (0.167)	1.308 (0.250)		
ϕ_{12}	0.136 (0.052)	-0.489 (0.398)		
ϕ_{21}			1.865 (0.008)	1.388 (0.113)
ϕ_{22}			-0.868 (0.003)	-0.732 (0.303)
Log Lik.	-114.669	-113.693	-159.331	-161.404
AIC	245.337	243.386	334.662	338.808
BIC	268.582	266.628	357.905	362.050

Note: The sample period is 1986Q3 to 2020Q1. Standard errors are shown in parentheses.

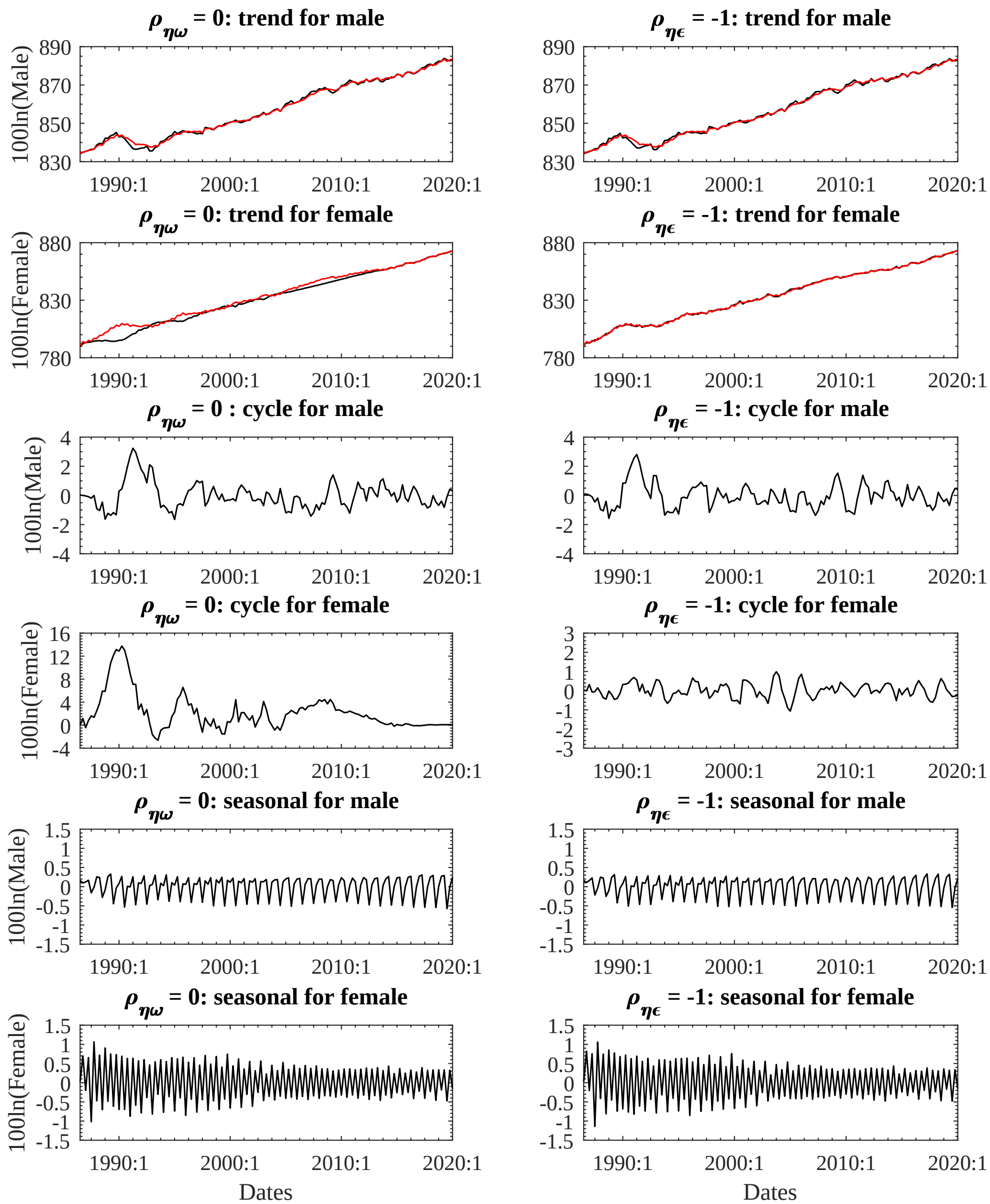

Figure 2: Estimated components of the univariate UC models for Australian male and female employments. The black lines are estimated components and the red lines are the employment data in natural logarithm times 100. The univariate model for the left column assumes zero correlation of trend and seasonal disturbances, i.e., $\rho_{\eta\omega} = 0$, and the univariate model for the right column restricts the correlation of trend and cycle disturbances to be -1, i.e., $\rho_{\eta\varepsilon} = -1$.

3.2 Bivariate analysis

Table 3 presents estimation results for four bivariate seasonal UC models for employment by gender in Australia. The first is the standard uncorrelated model, which allows nonzero disturbance correlations across variables only within each component, so that all cross-component correlations are assumed to be zero. The other three models show the common trend, common cycle and common seasonal models which allow cross-component correlations to be nonzero but impose restrictions as discussed in subsection 2.4 and specified in Table 1. Results for the other specifications discussed there are presented in Table 4. It may be noted that the number of restrictions imposed in each case (see Table 1) is substantially larger than the minimum of two required for identification.

Concentrating initially on the benchmark uncorrelated components and the common component models of Table 3, two specifications stand out in terms of the balance between goodness-of-fit and parameters estimated, namely in terms of information criteria values: these are the uncorrelated components and common cycle models, which are preferred by BIC and AIC respectively. Figure 3 compares the estimated components from these two models for the male and female employment series. Restricting the cross-component correlations to zero leads to the estimated trends for both series closely tracking the observed series, hence implying very small estimated cyclical variations. In contrast, the trend series extracted from the common cycle model are smooth and cyclical variation is evident. In particular, two downturns are detected during the 1990s, but with relatively little cyclical variation from early in the current century. These results for employment reflect the long period of growth experienced by the Australian economy since the 1990s[7]. Interestingly, and unlike results for the US (Hoynes, Miller and Schaller (2012), Guisinger (2020)), the estimate $\widehat{b} = 1.3$ for the common cycle model in Table 3 implies that cyclical variation in Australian employment is more marked for females than for men.

[7] World Bank data (https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=AU) shows positive annual GDP growth for Australia in each year from 1991 to 2019.

Table 3: Estimation results for bivariate UC models with uncorrelated and common components for male and female employment in Australia

Parameter	Uncorrelated	Common Trend	Common Cycle	Common Seasonal
Males:				
σ_{η_1}	0.544 (0.039)	0.631 (0.051)	0.489 (0.107)	0.741 (0.145)
σ_{ε_1}	0.025 (0.043)	0.296 (0.034)	0.041 (0.040)	0.268 (0.322)
σ_{ω_1}	0.015 (0.009)	0.020 (0.014)	0.011 (0.017)	0.016 (0.004)
$\rho_{\eta_1\varepsilon_1}$	0 (-)	-0.813 (0.058)	-0.170 (4.300)	-0.895 (0.173)
$\rho_{\eta_1\omega_1}$	0 (-)	-0.896 (0.245)	-0.719 (0.560)	-0.271 (0.821)
$\rho_{\varepsilon_1\omega_1}$	0 (-)	0.963 (0.189)	-0.399 (4.034)	0.599 (0.428)
Females:				
σ_{η_2}	0.635 (0.041)	$d \times \sigma_{\eta_1}$	0.681 (0.056)	1.459 (0.519)
σ_{ε_2}	0.138 (0.043)	0.831 (0.062)	$b \times \sigma_{\varepsilon_1}$	1.125 (0.508)
σ_{ω_2}	0.038 (0.011)	0.039 (0.011)	0.042 (0.025)	$a \times \sigma_{\omega_1}$
$\rho_{\eta_2\varepsilon_2}$	0 (-)	$\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_2\varepsilon_1}$	-0.917 (0.103)
$\rho_{\eta_2\omega_2}$	0 (-)	$\rho_{\eta_1\omega_2}$	0.519 (0.623)	$\rho_{\eta_2\omega_1}$
$\rho_{\varepsilon_2\omega_2}$	0 (-)	0.404 (1.413)	$\rho_{\varepsilon_1\omega_2}$	$\rho_{\varepsilon_2\omega_1}$
Cross-Series:				
$\rho_{\eta_1\eta_2}$	0.678 (0.056)	1 (-)	0.461 (0.207)	-0.478 (0.160)
$\rho_{\eta_1\varepsilon_2}$	0 (-)	-0.505(0.067)	$\rho_{\eta_1\varepsilon_1}$	0.785 (0.166)
$\rho_{\eta_1\omega_2}$	0 (-)	-0.556 (0.753)	-0.239 (0.865)	$\rho_{\eta_1\omega_1}$
$\rho_{\eta_2\varepsilon_1}$	0 (-)	$\rho_{\eta_1\varepsilon_1}$	-0.953 (1.256)	0.507 (0.280)
$\rho_{\eta_2\omega_1}$	0 (-)	$\rho_{\eta_1\omega_1}$	0.138 (1.121)	-0.191 (0.301)
$\rho_{\varepsilon_1\varepsilon_2}$	-0.995 (0.031)	0.911 (0.043)	1 (-)	-0.731 (0.249)
$\rho_{\varepsilon_1\omega_2}$	0 (-)	0.562 (1.122)	-0.658 (1.407)	$\rho_{\varepsilon_1\omega_1}$
$\rho_{\varepsilon_2\omega_1}$	0 (-)	0.795 (0.356)	$\rho_{\varepsilon_1\omega_1}$	0.067 (0.268)
$\rho_{\omega_1\omega_2}$	1.000 (0.001)	0.685 (0.244)	0.835 (0.445)	1 (-)
Others:				
β_1	0.366 (0.046)	0.386 (0.051)	0.345 (0.043)	0.386 (0.053)
β_2	0.605 (0.054)	$d \times \beta_1$	0.594 (0.061)	0.782 (0.142)
b			1.296 (1.134)	
a				2.269 (0.489)
d		0.948 (0.002)		
ϕ_{11}	0.181 (0.169)	1.655 (0.008)	1.872 (0.022)	0.037 (0.306)
ϕ_{12}	0.733 (0.154)	-0.713 (0.011)	-0.943 (0.029)	0.508 (0.171)
ϕ_{21}	-0.879 (0.313)	0.952 (0.012)		1.359 (0.013)
ϕ_{22}	-0.503 (0.312)	0.057 (0.012)		-0.354 (0.000)
Log Lik.	-257.512	-251.697	-248.193	-270.395
AIC	545.024	545.393	536.385	584.785
BIC	588.603	606.404	594.491	648.701

Note: The sample period is 1986Q3 to 2020Q1. Standard errors are shown in parentheses.

Table 4: Estimation results for other bivariate UC models for male and female employment in Australia

Parameter	Same cycle shock	Same trend shock	Perf-corr cycle shock	Perf-corr trend shock
Males:				
σ_{η_1}	1.144 (0.323)	1.418 (0.409)	0.478 (0.076)	4.291 (0.121)
σ_{ε_1}	0.877 (0.362)	1.589 (0.272)	0.171 (0.207)	4.157 (0.125)
σ_{ω_1}	0.016 (0.013)	0.018 (0.013)	0.017 (0.227)	0.022 (0.012)
$\rho_{\eta_1\varepsilon_1}$	-0.992 (0.008)	-0.942 (0.018)	-0.155 (1.311)	-0.994 (0.001)
$\rho_{\eta_1\omega_1}$	-0.807 (0.672)	0.517 (0.641)	-0.468 (2.335)	-0.913 (0.242)
$\rho_{\varepsilon_1\omega_1}$	0.744 (0.754)	-0.701 (0.615)	-0.650 (0.535)	0.879 (0.278)
Females:				
σ_{η_2}	1.207 (0.309)	σ_{η_1}	1.045 (1.050)	$d \times \sigma_{\eta_1}$
σ_{ε_2}	σ_{ε_1}	1.045 (0.332)	$b \times \sigma_{\varepsilon_1}$	9.536 (0.048)
σ_{ω_2}	0.041 (0.013)	0.041 (0.015)	0.043 (0.043)	0.043 (0.014)
$\rho_{\eta_2\varepsilon_2}$	$\rho_{\eta_2\varepsilon_1}$	$\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_2\varepsilon_1}$	$\rho_{\eta_1\varepsilon_2}$
$\rho_{\eta_2\omega_2}$	0.030 (0.533)	$\rho_{\eta_1\omega_2}$	0.510 (1.286)	$\rho_{\eta_1\varepsilon_2}$
$\rho_{\varepsilon_2\omega_2}$	$\rho_{\varepsilon_1\omega_2}$	-0.041 (0.962)	$\rho_{\varepsilon_1\omega_2}$	0.435 (0.660)
Cross-Series:				
$\rho_{\eta_1\eta_2}$	0.866 (0.074)	1 (-)	0.385 (0.199)	1 (-)
$\rho_{\eta_1\varepsilon_2}$	$\rho_{\eta_1\varepsilon_1}$	-0.919 (0.086)	$\rho_{\eta_1\varepsilon_1}$	-0.999 (0.000)
$\rho_{\eta_1\omega_2}$	-0.464 (0.474)	0.053 (0.913)	0.072 (1.465)	-0.455 (0.649)
$\rho_{\eta_2\varepsilon_1}$	-0.912 (0.063)	$\rho_{\eta_1\varepsilon_1}$	-0.971 (0.281)	$\rho_{\eta_1\varepsilon_1}$
$\rho_{\eta_2\omega_1}$	-0.497 (0.898)	$\rho_{\eta_1\omega_1}$	0.495 (1.270)	$\rho_{\eta_1\omega_1}$
$\rho_{\varepsilon_1\varepsilon_2}$	1 (-)	0.998 (0.013)	1 (-)	0.997 (0.001)
$\rho_{\varepsilon_1\omega_2}$	0.353 (0.526)	-0.041 (0.947)	-0.527 (1.762)	0.473 (0.652)
$\rho_{\varepsilon_2\omega_1}$	$\rho_{\varepsilon_1\omega_1}$	-0.728 (0.646)	$\rho_{\varepsilon_1\omega_1}$	0.894 (0.266)
$\rho_{\omega_1\omega_2}$	0.744 (0.286)	0.580 (0.434)	0.751 (0.611)	0.630 (0.355)
Others:				
β_1	0.378 (0.082)	0.289 (0.113)	0.370 (0.081)	0.693 (0.061)
β_2	0.620 (0.086)	0.740 (0.497)	0.668 (0.062)	1.155 (0.121)
b			3.783 (3.330)	
d				2.147 (0.063)
ϕ_{11}	1.254 (0.101)	1.024 (0.002)	1.784 (0.070)	0.978 (0.009)
ϕ_{12}	-0.434 (0.127)	-0.034 (0.019)	-0.804 (0.029)	-0.018 (0.011)
ϕ_{21}	1.313 (0.100)	1.326 (0.001)	1.642 (0.012)	0.961 (0.001)
ϕ_{22}	-0.462 (0.109)	-0.322 (0.013)	-0.648 (0.008)	0.021 (0.003)
Log Lik.	-251.868	-252.705	-247.139	-247.142
AIC	545.735	547.409	538.279	538.285
BIC	606.746	608.42	602.195	602.201

Note: The sample period is 1986Q3 to 2020Q1. Standard errors are shown in parentheses.

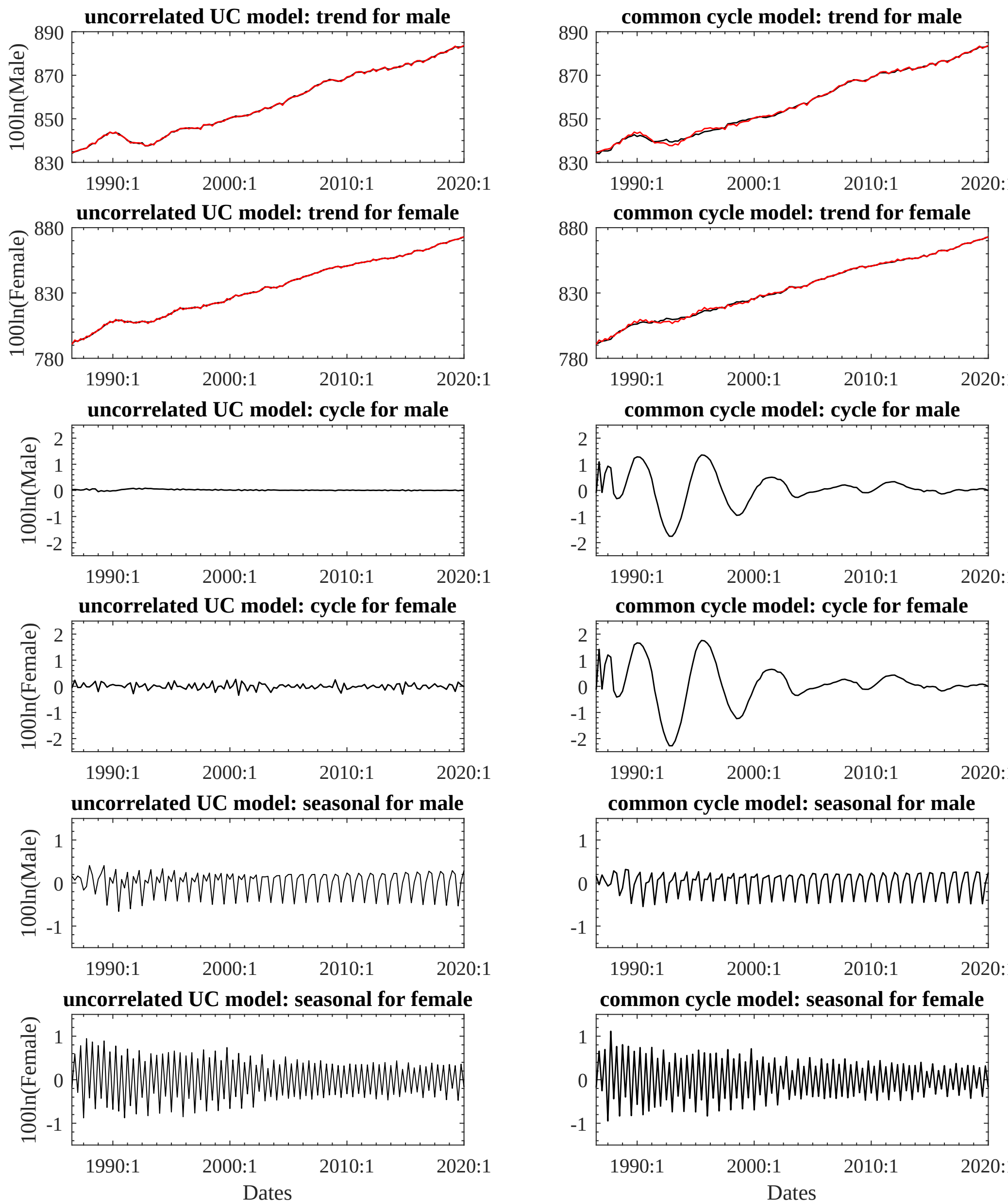

Figure 3: Comparison of the estimated trend, cycle and seasonal components for an uncorrelated UC model and a common cycle model for Australian male and female employment. The black lines are estimated components and the red lines are employment values in natural logarithm times 100.

It is also notable that the bivariate common cycle model yields much smoother cyclical components than any of the univariate UC models in Figure 2, indicating the value of combining information available in the two series alongside a flexible covariance structure. Also note that some (albeit fairly subtle) differences can be seen in the estimated seasonal component for each series across the models of Figures 2 and 3.

It is also useful to discuss the results of the other specifications in Tables 3 and 4. We have already commented above that male and female employment in Australia appear to exhibit different deterministic trends over time, and hence it is unsurprising that the common trend model of Table 3 is not a preferred specification. While the less restricted version of the perfectly correlated trend shock model in Table 4 yields improved values for the information criteria, the common cycle model is still preferred to this specification and also to the same trend shock model of that table. In the light of the estimated seasonal patterns for the two series across a range of specifications, it is also unsurprising that the common seasonal model of Table 3 leads to relatively poor information criteria values. Further, note that the common cycle model is preferred to the perfectly correlated cycle shock and the same cycle shock models (Table 4)[8].

In summary, having examined a range of specifications, the common cycle model for gender employment in Australia is preferred. It produces plausible outcomes for trend, cycle and seasonal components for males and females and also gives the lowest AIC value across the seven bivariate models considered. The results suggest that males and females do not have the same seasonality. Explanations might be the sectors in which females work or that they prefer part-time to full-time.

[8]The figures in the appendix (Figures A.1 - A.4) correspond to the models of Table 4.

4 Conclusion

Multivariate analysis of economic time series can throw important light on underlying economic phenomena, including trend, cyclical and seasonal movements. In order to analyze such movements when they are potentially correlated, a correlated multivariate unobserved components model is required. Although previously considered in a univariate context, to the best of our knowledge the present paper is the first to study identification conditions for a multivariate trend-cycle-seasonal model with correlated shocks. Although restrictions are required to deliver identification, we believe that forms of cross-equation restrictions that we study (including common trends, common cycles and common seasonality) are intuitive and allow the approach to be applied in a variety of real-world situations.

The approach is illustrated by an application to quarterly aggregate male and female employment in Australia. Although a range of specifications is considered, including common trend, common seasonality and the uncorrelated component model, the common cycle specification is preferred. Indeed, graphical and univariate analyses also point to a common cycle as the most plausible form of restriction to be imposed, with evidence of distinct gender-based trend and seasonal patterns.

References

Birch, E. and A. Preston (2020). The Australian labour market in 2019. *Journal of Industrial Relations 62*, 341–364.

Cecchetti, S. G. and A. K. Kashyap (1996). International cycles. *European Economic Review 40*, 331–360.

Clark, P. K. (1989). Trend reversion in real output and unemployment. *Journal of Econometrics 40*, 15–32.

Dufour, J. M. and D. Pelletier (2021). Practical methods for modelling weak VARMA processes: Identification, estimation and specification with a macroeconomic application. *Journal of Business and Economic Statistics*. forthcoming.

Dungey, M., J. P. Jacobs, J. Tian, and S. van Norden (2013). On the correspondence between data revision and trend-cycle decomposition. *Applied Economics Letters 20*, 312–315.

Dungey, M., J. P. Jacobs, J. Tian, and S. van Norden (2015). Trend in cycle or cycle in trend? New structural identifications for unobserved components models of U.S. real GDP. *Macroeconomic Dynamics 19*, 776–790.

Fleischman, C. A. and J. M. Roberts (2011). From many series, one cycle: Improved estimates of the business cycle from a multivariate unobserved components model. Finance and Economics Discussion Series 2011-46, Federal Reserve Board, Washington D.C., USA.

Ghysels, E. and D. R. Osborn (2001). *The Econometric Analysis of Seasonal Time Series*. Cambridge, U.K.: Cambridge University Press.

Guisinger, A. Y. (2020). Gender differences in the volatility of work hours and labor demand. *Journal of Macroeconomics 66*, 103254. forthcoming.

Harvey, A. C. (1989). *Forecasting, Structural Time Series Models and the Kalman Filter*. Cambridge, UK: Cambridge University Press.

Harvey, A. C. and T. M. Trimbur (2003). General model-based filters for extracting cycles and trends in economic time series. *Review of Economics and Statistics 85*, 244–255.

Hindrayanto, I., J. P. Jacobs, D. R. Osborn, and J. Tian (2019). Trend-cycle-seasonal interactions: identification and estimation. *Macroeconomic Dynamics 23*, 3163–3188.

Hoynes, H., D. L. Miller, and J. Schaller (2012). Who suffers during recessions? *Journal of Economic Perspectives 26*, 27–48.

Krane, S. and W. Wascher (1999). The cyclical sensitivity of seasonality in U.S. employment. *Journal of Monetary Economics 44*, 523–553.

Ma, J. and M. E. Wohar (2013). An unobserved components model that yields business and medium-run cycles. *Journal of Money, Credit and Banking 45*, 1351–1373.

Matas-Mir, A. and D. R. Osborn (2004). Does seasonality change over the business cycle? An investigation using monthly industrial production series. *European Economic Review 48*, 1309–1332.

McElroy, T. (2017). Multivariate seasonal adjustment, economic identities and seasonal taxonomy. *Journal of Business and Economic Statistics 35*, 611–625.

Morley, J. C. (2007). The slow adjustment of aggregate consumption to permanent income. *Journal of Money, Credit and Banking 39*, 615–638.

Morley, J. C., C. R. Nelson, and E. Zivot (2003). Why are the Beveridge-Nelson and unobserved-components decompositions of GDP so different? *The Review of Economics and Statistics 85*, 235–243.

Sinclair, T. M. (2009). The relationships between permanent and transitory movements in U.S. output and the unemployment rate. *Journal of Money, Credit and Banking 41*, 529–542.

Stock, J. H. (2013). Comments on Unseasonal Seasonals? *Brookings Papers on Economic Activity Fall*, 111–119.

Trenkler, C. and E. Weber (2016). On the identification of multivariate correlated unobserved components models. *Economics Letters 138*, 15–18.

Wright, J. H. (2013). Unseasonal seasonals? (Including comments and discussion). *Brookings Papers on Economic Activity Fall*, 65–126.

A Additional results

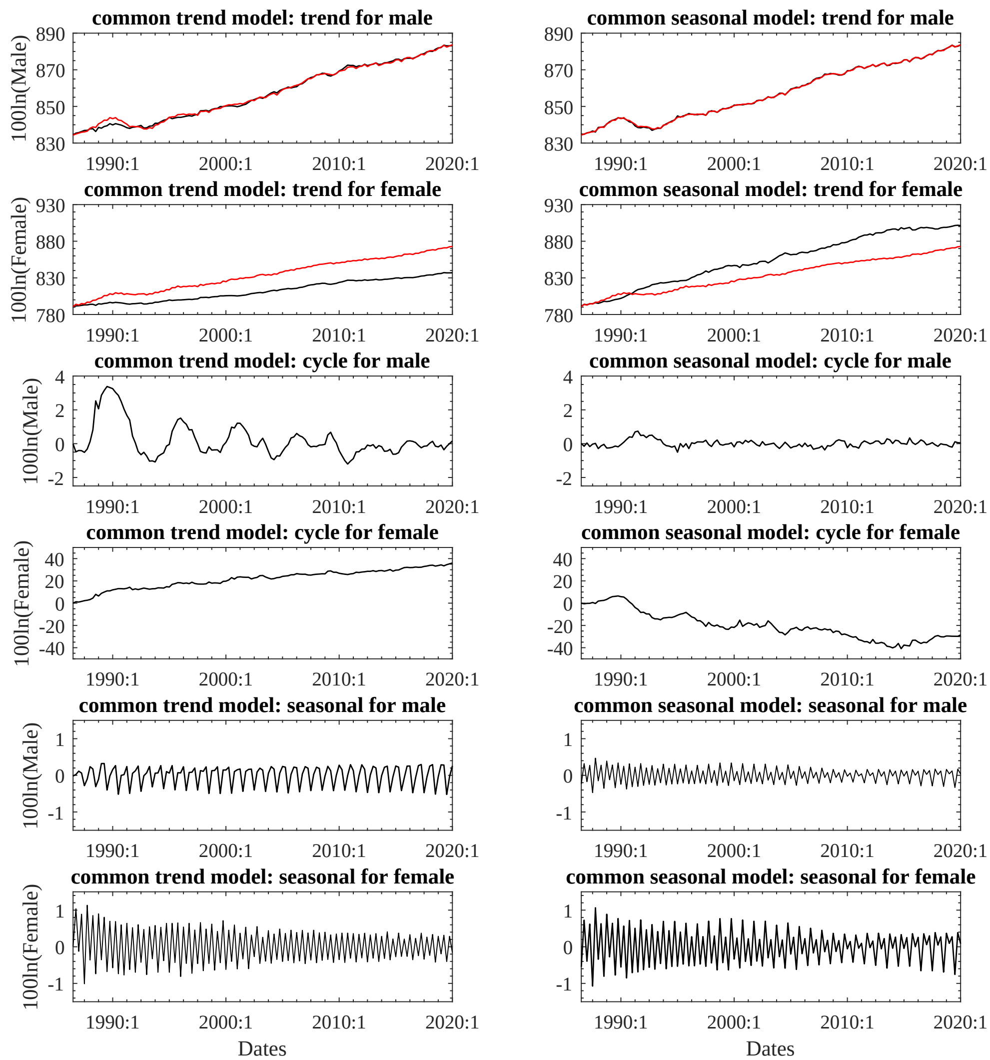

Figure A.1: A comparison of estimated trend, cycle and seasonal components for a common trend model and a common seasonal component model. The Black lines are estimated components and the red lines are employment values in natural logarithm times 100.

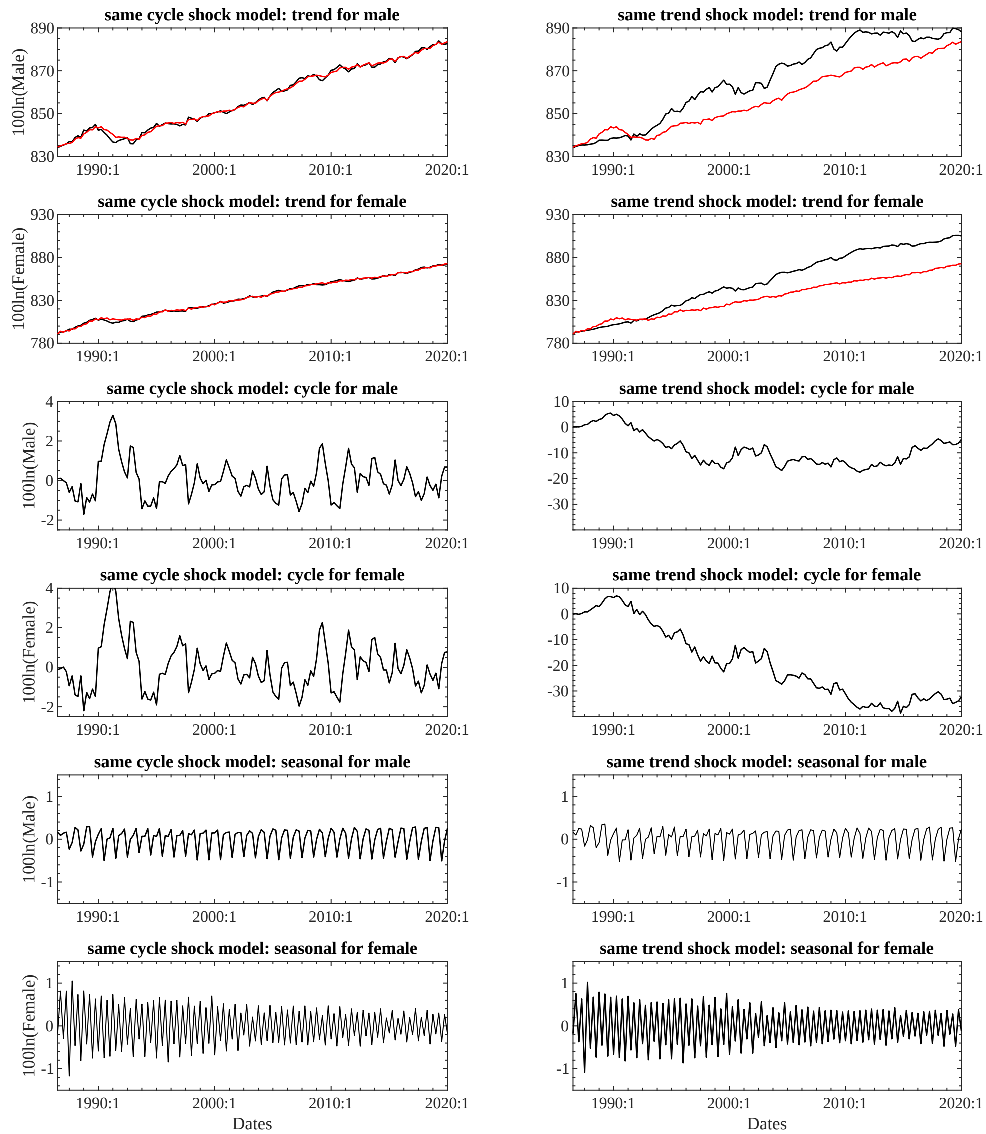

Figure A.2: A comparison of estimated trend, cycle and seasonal components for the model with the same cycle shock and the model with the same trend shock. The Black lines are estimated components and the red lines are employment values in natural logarithm times 100.

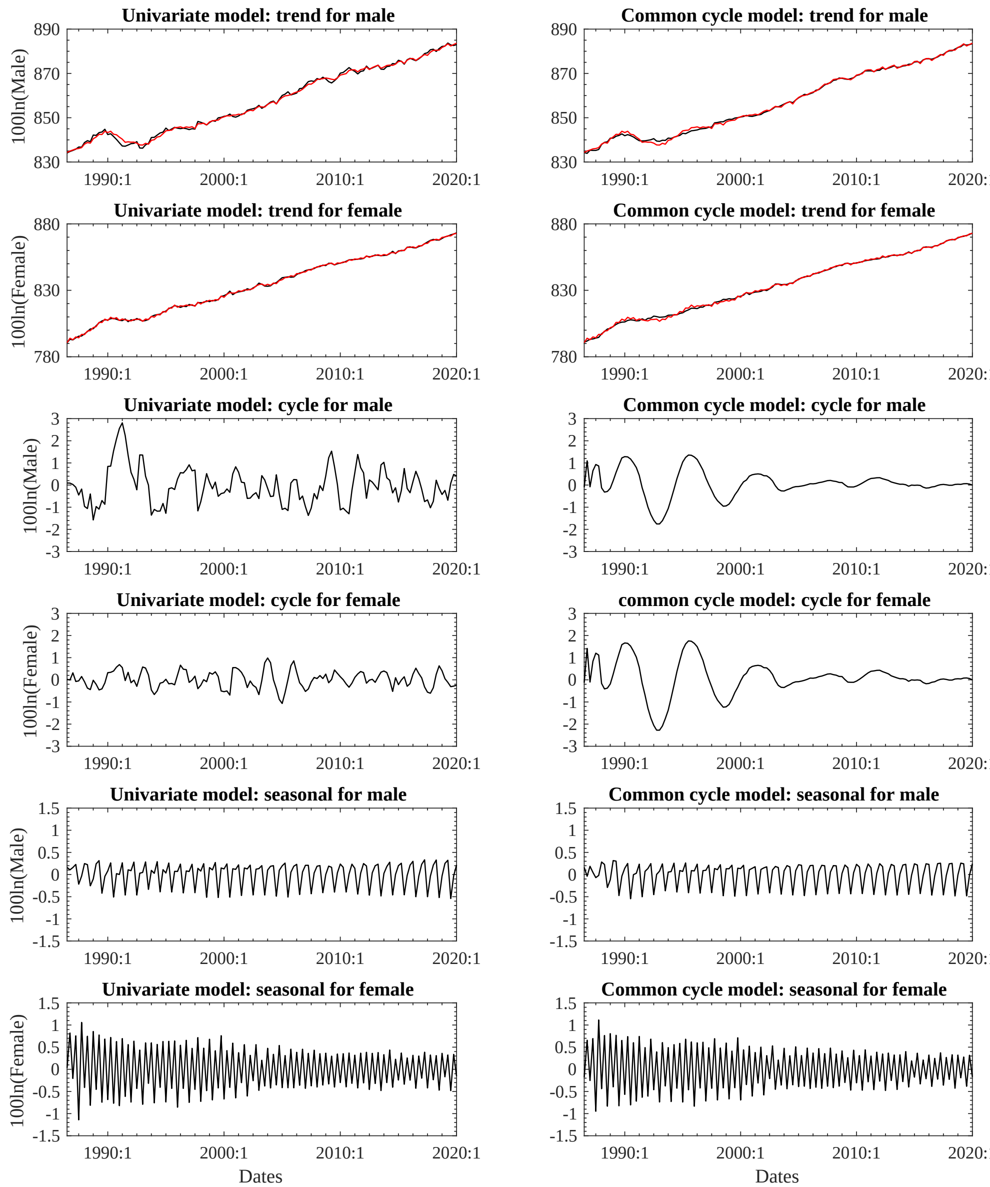

Figure A.3: A comparison of estimated trend, cycle and seasonal components between the univariate models and a bivariate model, in which male and female employment share a common cycle component. The Black lines are estimated components and the red lines are employment values in natural logarithm times 100. In the univariate model the correlation between trend and cycle shocks is assumed equal to -1.

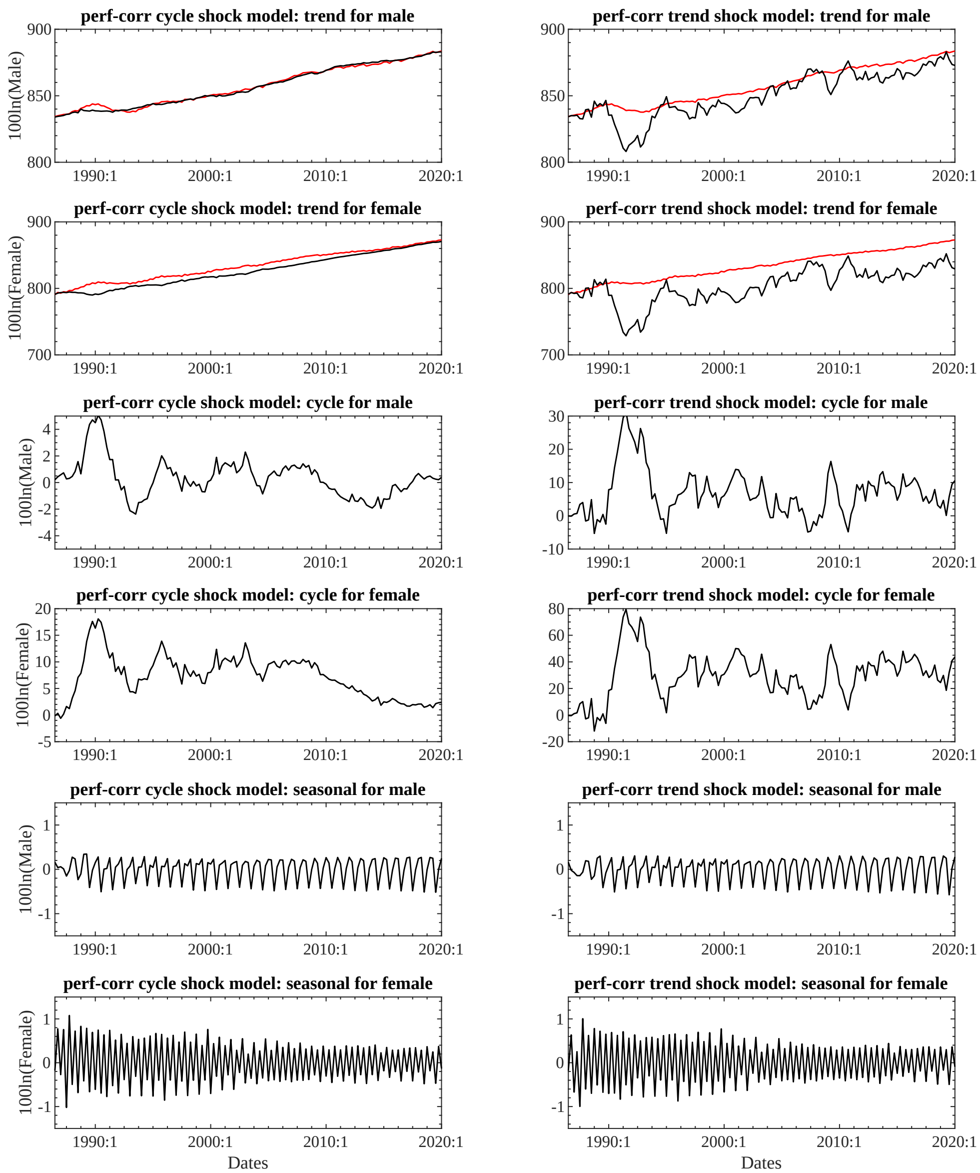

Figure A.4: A comparison of estimated trend, cycle and seasonal components between the bivariate model with perfectly correlated cycle shocks and the bivariate model with perfectly correlated trend shocks. The Black lines are estimated components and the red lines are employment values in natural logarithm times 100.