Scattering of two particles in a one-dimensional lattice incorporating multi-band effects

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Synopsis The presence of a lattice potential can greatly modify the scattering behavior of two interacting particles. The particles are energetically confined to bands. However, interactions can excite the particles to energetically closed bands. Bound dimers attached to these inaccessible bands can create scattering resonances in analogy to the confinement induced resonances that occur in waveguide systems.

To say that ultracold atomic systems have been the focus of a great deal of theoretical and experimental interest over the past two decades may be somewhat of an understatement. With a great deal of experimental control over nearly every aspect of the system, these gases can be used as a test bed and model for a great variety of quantum-mechanical behaviors and as a proxy to explore new regimes. By incorporating optical lattices, they can also be used to explore a variety of solid-state systems with the atoms playing the role of electrons. In these scenarios, a detailed knowledge of the inter-particle interactions is required to predict the behaviors and control the system.

The simplest approach to model a an ultracold gas loaded into an optical lattice is to transform the system into a set of Wannier states. If the system is restricted a single conduction band (only one Wannier state on each lattice site), this approximation results in a Hubbard model in which atoms can “hop” from site to site, and - if atom-atom interactions are short range - interact via an on-site, two-body interaction. However, when two particles scatter off each other on a single lattice site, they can virtually scatter to energetically inaccessible bands. These virtual excitations can create scattering resonances and modify the effective on-site interaction parameters. The effects of these resonances have been studied in the low energy regime and can result in scattering resonances due to a dimer bound state attached to excited bands by Fedichev *et al.* [1]. The resulting effective interactions have also been predicted in the zero relative-momentum limit by Cui *et al.* [2].

In the work presented here, we examine two-particles in a one-dimensional lattice shown schematically in Fig. 1. The system ca be discretized by transforming into the Wannier basis. In the limit of deep lattice sites, the Wannier states are localized to each lattice site and motion of the atoms through the lattice is described by a band index dependent nearest-neighbor hopping.

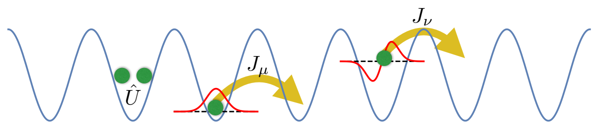

Figure 1. Schematic of the multi-band lattice model.

Asymptotically, the two particles are relegated to remain in bands that are energetically accessible. However, when the particles occupy the same site, they can virtually scatter, via an on-stie interaction matrix $\hat{U}$, into bands that are energetically closed. We incorporate this virtual scattering by solving the Lippmann-Schwinger equation for the reactance matrix (K-matrix) using a lattice Green’s operator. The resulting formula for the K-matrix for open band scattering bears a striking similarity to that which arrises from channel closing formulas in standard multi-channel scattering theory. We then apply this formula for two-body scattering in the lowest and first excited bands within a two band approximation. Within this approximation, virtual scattering into closed bands can create scattering resonances in the presence of bound states attached to closed bands in analogy to Feshbach or confinement induced resonances.

References

[1] P. O. Fedichev *et al.* 2004 *Phys. Rev. Lett.* **92** 080401

[2] X. Cui *et al.* 2010 *Phys. Rev. Lett.* **104** 153201

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