The investigation of a wake potential of protons channeled in an axial regime in ionic crystals

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Synopsis There are calculated the time dependent electric dipole moments of crystal medium ions, induced under the influence of electric and magnetic fields of protons channeling in an axial regime in ionic crystals. It is shown that a summary non-stationary potential of this system of dipoles leads to formation of a wake potential which essentially influences the channeling regime.

The given paper which is the continuation of [1, 2], deals with the consideration of an axial channeling of protons of various energies in different ionic crystals. It is shown that due to the polarization of the medium by a moving proton, while taking into account the spatial and temporal dispersion, are induced non-stationary dipole moments that lead to formation of wake potential.

The calculation of the wake potential was conducted in following steps.

1. The intensities of electric and magnetic fields of proton moving at the velocity \( v = v_0 \) and in quantum state of axial channeling \( \psi_{n_{z},l}(\tilde{p}) \), i.e. characterized by the density of the electric charge

\[
\rho_{n_{z},l}(\tilde{p}) = e \psi_{n_{z},l}(\tilde{p}) \delta(z - vt),
\]

is calculated in accordance with [3]:

\[
\vec{E}_{n_{z},l}(\tilde{r},t) = \frac{e}{\gamma^{2}} \int \psi_{n_{z},l}(\tilde{\xi})^{2} \tilde{R} \left( \tilde{r}, \tilde{\xi},t \right) \left( R^{*3} \left( \tilde{r}, \tilde{\xi},t \right) d^{3}\tilde{\xi},
\]

\[
\vec{H}_{n_{z},l}(\tilde{r},t) = \left[ \vec{v}, \vec{E}_{n_{z},l}(\tilde{r},t) \right] / c, \tilde{R} \left( \tilde{r}, \tilde{\xi},t \right) = \tilde{r} - \tilde{\xi}.
\]

Here \( \gamma \) – a Lorentz-factor, \( \tilde{\xi} = \tilde{p}_{0} + vt \tilde{e}_{z} \) and \( \tilde{r} \) – the coordinates of a moving proton and observation point, \( R^{*3} \left( \tilde{r}, \tilde{\xi},t \right) = |\tilde{r} - \tilde{\xi}|^{2} / \gamma^{2} + (z - vt)^{2} \).

2. The induced electric dipole moment \( \vec{p}_{n_{z},l}(\tilde{r}_{n_{z}},t) \) of a separate crystal ion in the point \( \tilde{r}_{n_{z}} = n_{z} a_{z} \tilde{e}_{z} \) (here \( a_{z} \) – the period of a crystal lattice along \( z \) axis; \( n_{z} = 0, \pm 1, \pm 2, \ldots \)), under the influence of proton in the state \( \psi_{n_{z},l}(\tilde{p}) \), is calculated by means of the following oscillator type equation:

\[
\vec{p}_{n_{z},l}(\tilde{r}_{n_{z}},t) + g \vec{p}_{n_{z},l}(\tilde{r}_{n_{z}},t) + \omega_{0}^{2} \vec{p}_{n_{z},l}(\tilde{r}_{n_{z}},t) =
\]

\[
= \frac{e^{2}}{m} \int d\tilde{r} \kappa(\tilde{r} - \tilde{r}_{n_{z}}) \left[ \vec{E}_{n_{z},l}(\tilde{r},t) + \frac{1}{c} \left[ \vec{v}, \vec{H}_{n_{z},l}(\tilde{r},t) \right] \right],
\]

where \( m \) – the proton rest mass, \( \kappa(r) \) – the density of electron distribution in an ion, \( g \) – the attenuation constant that connected with electronic and nucleus oscillations, \( \omega_{0} \) – eigen frequency of oscillator.

3. A wake potential induced by the system of these electric non-stationary dipoles \( \vec{p}_{n_{z},l}(\tilde{r}_{n_{z}},t) \), is calculated by means of the following expression:

\[
\Phi(\tilde{r},t) = \sum_{n_{z},l} \left[ \vec{p}_{n_{z},l}(\tilde{r}_{n_{z}},t) \right] \left( \tilde{r} - \tilde{r}_{n_{z}} \right) / \left( \tilde{r} - \tilde{r}_{n_{z}} \right)^{3} \] \( \omega_{n_{z},l} \),

where \( \omega_{n_{z},l} \) – the probability of proton population in quantum state \( \psi_{n_{z},l}(\tilde{p}) \) of axial channeling motion.

Note that the calculations of the wake potentials \( \Phi(\tilde{r},t) \) for all ionic crystals agree quite well with the calculations carries out in accordance with the theory on a wake effect in nonmetallic foils proposed in [4].

References


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