TRANSPORT ELECTRIFICATION FOR SUSTAINABLE URBAN NETWORKS

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ABSTRACT

Although transportation has long been recognised as one of the critical factors for socio-economic development, it is responsible for many global greenhouse gas emissions and significant pollutants that cause severe health problem, especially in urban areas. Besides, the rapid development of transportation also brings great concern about energy security. To reduce the transportation system's side effects, Electric vehicles (EVs) have emerged as a promising solution toward sustainable transportation due to their positive impact on environmental issues and energy crisis. However, the adoption of EVs is still minimal compared to conventional gasoline vehicles due to the lack of appropriate charging infrastructure. Furthermore, the electrification of transportation may cost a significant amount of money and result in more congestion (i.e., en-route and charging congestion) due to EVs' routing and charging behaviours. Having these concerns in mind, in this study, we focus on answering the questions of where and how to deploy the charging facilities to promote the widespread adoption of EVs and improve the system performance in the presence of EVs. The objective is to minimise the investment cost of charging infrastructure and the en-route and charging congestion by capturing travellers' routeing choice behaviours with stochastic demand and driving range. The problem is first formulated as a bi-level optimisation problem to capture the mutual interaction between planning decision and traffic flow pattern on the network and then solved by a meta-heuristic. Finally, the proposed framework is tested through the numerical test, and some managerial insights into the facility planning and system performance also be provided.

INTRODUCTION

Although transportation has long been perceived as the critical component for socio-economic development, this sector is responsible for a third of total global greenhouse gas emissions and significant pollutants that cause a severe health problem. In New Zealand, road transportation emissions in 2018 were up 2.0 per cent from 2017 and up 101.6 per cent from 1990 (StatsNZ, 2020). According to the statistics, on-road vehicles made up 42.6 per cent of all carbon dioxide emissions in 2018. Moreover, the significant increase in transport demand raises a big concern for energy security. In the US, transportation is responsible for 29 per cent of the total energy consumption, with 92 per cent related to fossil fuel (Ngo, et al., 2020). People are starting to shift in favour of Electric Vehicles (EVs) to mitigate the adverse effects of transportation, which marks a new transportation electrification era.

To be prepared for the upcoming electrification revolution of transportation, billions of dollars in subsidies for charging infrastructure have been provided by governments and automakers worldwide (IEA, 2019). In May 2016, the New Zealand Government announced the EV Programme, which aimed to help develop New Zealand’s EV market by reducing some of the barriers and investigating ways to encourage people to buy EVs. The Programme aims to increase the number of electric vehicles in New Zealand to have 64,000 electric vehicles on our roads by 2021. Besides, New Zealand already boasts more than 80 per cent of renewable electricity generation. Electrification of the transport sector, accounting for 36 per cent of energy use in New Zealand, will further drive the decarbonisation effort. The country aims to have an entire zero-emission bus fleet by 2040 (SmartCitiesWorld.net, 2020).

There are two necessary charging facilities currently deployed to serve EVs' users, including low-power (level 1 and level 2 modes) and DC rapid charging (level 3 mode). While the low charging modes require several hours for a full recharge, the fast charging mode can handle the urgent need for charging in less than 10 minutes with much higher installation costs (Wu & Sioshansi, 2017). In New Zealand, there are 144 DC rapid charging stations on the North Island and a further 65 on the South Island. However, EVs' widespread adoption is still limited due to their limited driving ranges, long charging time, and insufficient charging facilities. This paper focuses on the fast-charging infrastructure for personal or private EVs due to their significant role in mitigating travellers' range anxiety (Guo, et al., 2018).

The charging infrastructure deployment process can be stated as a chicken-and-egg dilemma. Although investment decisions of where to deploy facilities are costly and affect a long-time horizon, the charging stations need to be provided before observing the actual demand. It emphasises the stochastic nature of the charging facilities planning problem. Therefore, we put our effort to capture three primary sources of uncertainty in the present paper: travellers' demands, EVs' driving ranges and route choice behaviours.

Finally, the electrification of transportation and infrastructure deployment also result in changes in traffic flow. Although more EVs can bring a cleaner and more energy-efficient transportation system, it sometimes may cause more congestion over the network (i.e., en-route and charging congestion) (Tran, et al., 2020). Optimising the charging locations assuming that the flow pattern remains unchanged may lead to an unreliable solution or a deterioration in network performance due to some re-routing of traffic responding to the changing of charging locations. Therefore, it is crucial to develop a systematic framework for deploying charging infrastructure to minimise the capital cost and reduce the congestion and environmental cost with consideration of stochastic driving range, charging congestion, and the mutual interaction between charging locations and traffic flow pattern.

Having the motivation mentioned above and research gaps in mind, the paper's overall objective is to optimally deploy fast-charging stations to minimise the expected system cost. The expected system cost consists of the infrastructure investment on the charging stations, the expected monetary value of total travel time and the environmental cost. Furthermore, the drivers' route choice behaviour under stochastic demand and driving range is recognised and incorporated into the model.

MODEL FORMULATION

The EVs-charging location problem can be categorized into node-based, flow-based, and equilibrium-based approaches depending on the charging demand pattern and route choice behaviours (Shen, et al., 2019). In this study, we adopt the equilibrium-based approach to avoid the deterioration in network performance due to the mutual interaction of re-routing behaviours and the charging locations decision. The problem is formulated as a bi-level optimization program with two different levels, as shown in Figure 1. The first-level decision-maker is the upper-level decision-maker (leader) and the second level decision-maker is the lower-level decision-maker (follower).

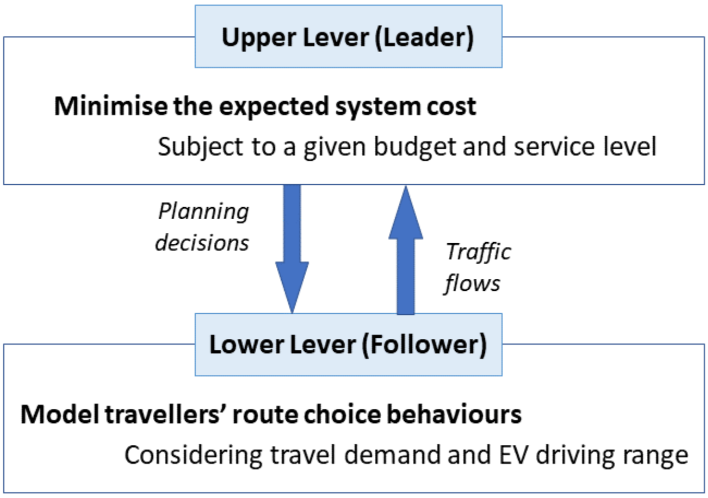

Figure 1. The bi-level framework

At the upper level, the planner decides where and how many chargers to be deployed to minimize the expected system cost, including the charging infrastructure investment cost, the expected travel cost, and the environmental cost, according to a budget constraint and service level. The output of the upper-level is the planning decisions on the charging infrastructure.

At the lower level, the equilibrium traffic flow is determined by solving a multi-class probit-based SUE model with Poisson demand (probit-based SUE-P) and multinomial conditional route choice. The traffic flow then will come back to the upper-level as an input.

The proposed bi-level optimization model is formulated in (1) – (7). The used notations for the model are listed in Table 1.

Symbol	Definition
K	Set of nodes, $k \in K$
A	Set of links, $a \in A$
W	Set of all O-D pairs, $w \in W$
N	Set of vehicle classes, $n \in N$
P^w	Set of all path p between O-D pair $w \in W,\ p \in P^w$
x_k	Whether a charging station is located at location k or not
u_k	Number of chargers placed at location k
$y_p^{w,n}$	Whether path p between pair w is feasible for vehicle class n or not
$q^{w,n}$	Mean of travel demand of vehicle class n between O-D pair w
$f_p^{w,n}$	Mean traffic flow of vehicle class n on path p between O-D pair w

q_a	Mean of aggregate traffic flow on link a
t_a	Mean of travel time on link a
t_p^w	Mean travel time on path p between O-D pair w
$l_{s,p}^w$	Length of sub-path s on path p between O-D pair w
D^n	Random driving range of vehicle class n
$\mathbb{G}_{s,p}^{w,n}$	Probability that driving range D^n is smaller than the length of sub-path s of path p

Table 1. Notations

$$\min_{(\boldsymbol{X},\boldsymbol{f})} \mathrm{PI}(\boldsymbol{X},\boldsymbol{f}) = \sum_{k \in K} (c_k\, x_k + h_k\, u_k) + v_1 \left(\sum_{w \in W} \sum_{p \in P^w} \sum_{n \in N} t_p^w\, f_p^{w,n} + \sum_{k \in K} W_k \lambda_k \right) + v_2 \sum_{a \in A} q_a^g e_a^g \tag{1}$$

Subject to

$$\boldsymbol{X}(\boldsymbol{x},\boldsymbol{u},\boldsymbol{y}) \in \Omega \tag{2}$$

$$\boldsymbol{f} \in \operatorname{argmin} \left\{ \mathrm{Z}(\boldsymbol{f}) = -\sum_{w \in W} \sum_{n \in N} q^{w,n} \mathbb{E}\left[min_{p \in P^w} \{t_p^w\} | t^w(\boldsymbol{f}) \right] + \sum_{a \in A} q_a t_a(q_a) - \sum_{a \in A} \int_0^{q_a} t_a(q_a) d_q : \boldsymbol{f} \in \Theta \right\} \tag{3}$$

$$x_k = \{0, 1\} \tag{4}$$

$$u_k \geq 0 \tag{5}$$

$$y_p^{w,n} = \{0, 1\} \tag{6}$$

$$0 \leq f_p^{w,n} \leq \mathrm{M} y_p^{w,n} \tag{7}$$

The first term of the objective function is the capital cost incurred by installing charging stations and placing the chargers at each station. The station installation costs, c_k may include site acquisition, utility provision, permitting, project management, etc., which can be estimated based on the average cost in a certain particular area. In contrast, the charger unit cost, h_k can be found varied due to the providers (Ghamami, et al., 2019).

The second term of the objective function is the expected travel cost calculated by the monetary value of time, v_1. The total travel time includes the expected en-route travel time and expected charging time at charging stations which are resulted from the route choice behaviour of travellers. The expected waiting time at charging station k, W_k can be obtained by adopting the queuing theory with the arrival rate of EVs λ_k (Jung, et al., 2014).

The final term of the objective function is the environmental cost. The on-road vehicles have been known as a significant contribution to the air pollution, including carbon monoxide (CO), volatile organic compounds (VOC), nitrogen oxides NO_x and particulate matter (PM). In fact, on-road vehicles are responsible for most of CO emissions in the air (Yin, et al., 2014). In this paper, therefore, we consider CO as an indicator of the level of air pollution generated by GVs while EVs can be seen as zero-emission vehicles. The total amount of traffic emissions then can be calculated by the product of the average amount of traffic emissions, e_a^g and the GVs traffic flow on the network, q_a^g and converted into the monetary value by CO unit cost, v_2.

Constraint (2) presents the feasible space of the upper level, which are defined by Constraints (8) – (11). Constraint (8) entails the maximum number of charging stations to be located (according to a given budget). The relationship between charging locations and feasible paths is shown as in Constraint (9) and (10).

In comparison with GVs, EVs' users choose the route to minimize their perceived travel times and have to consider the feasibility of the selected route. EVs' drivers are assumed to be willing to take a trip if the probability of running out of fuel during this trip is under a maximum acceptable risk threshold, α. In Constraint (11), we imply a chance constraint on the driving range to capture the problem's stochastic nature. It is worth to note that in this study, EVs' drivers are allowed to have multiple en-route recharging with the stochastic charging demand.

$$u_k \leq m\, x_k \tag{8}$$

$$\left[D^n - \max\left(l_{s,p}^w\right)\right] y_p^{w,n} \geq 0 \tag{9}$$

$$y_p^{w,n} > \frac{D^n - \max\left(l_{s,p}^w\right)}{D^n} \tag{10}$$

$$\mathbb{G}_{s,p}^{w,n} \leq \alpha + \left(1 - y_p^{w,n}\right) \tag{11}$$

Constraints (4) – (6) specify the binary decision vectors **x**, **y** and non-negative vector **u**. Constraint (7) is the side-constraints on feasible path flow due to limited feasible paths in which M is a large positive number.

Constraint (3) is the lower level defined as an equivalent minimization, where Ɵ is the feasible space of path flow solutions defined by the flow conservation and following constraints (Sheffi, 1985). In our study, the equilibrium flows $\boldsymbol{f}$ is corresponded to each locating solution $\boldsymbol{X}, \boldsymbol{f} = \boldsymbol{f}(\boldsymbol{X})$. The travel demand between O-D pair w is assumed to be Poisson, $Q^w \sim Poisson(q^w)$. Consequently, the resulting path flows of each vehicle class n between O-D pair w follow independent Poisson distribution, $F_p^{w,n} \sim Poisson\left(f_p^{w,n}\right)$ (Nakayama & Watling, 2014) with the mean path flow which is the solution to the following equivalent fixed-point problem.

$$f_p^{w,n} = q^{w,n}\, Pr_p^{w,n}(t^w(\boldsymbol{f})) \tag{12}$$

$Pr_p^{w,n}$ denotes the probability that vehicle class n choose path p between O-D pair w.

SOLUTION METHOD

The proposed bi-level framework is strong NP-hard due to the binary-type decision variables, stochasticity, and intractable structure. Therefore, it is non-viable to find an exact global solution to the problem. Instead, the meta-heuristic approach, such as Genetic Algorithm, Hill Climbing, Simulated Annealing, etc. usually might be applied to obtain a good solution in a reasonable amount of time. The comparison of such different algorithms is beyond the scope of this study. In the present paper, we have adopted a relatively new approach based on the Cross-Entropy Method due to its robustness and insensitivity to the initial solutions.

The Cross-Entropy Method (CEM) was initially proposed by Rubinstein & Kroese (2004) as an adaptive variance minimization algorithm for estimating rare events probabilities on stochastic networks. Eventually, this method was adopted to solve both static and noisy combinatorial optimization problems effectively, including network design problems in the transportation field

(Ngoduy & Maher, 2012; Abudayyeh, et al., 2021). In general, the CEM consists of two steps:

1. Generating the sample of candidate solutions using a given parameterized distribution;

2. Updating the sampling distribution parameters to steer the problem towards the optimal solution over subsequent iterations.

The details of CEM-based algorithm applied to solve deterministic fast-charging facility deployment problem as the bi-level program has been proposed in the study of Tran, et al. (2020). In the present paper, we extend this approach to consider the environmental cost, stochastic driving range, stochastic charging demand and charging congestion. Because charging infrastructure would probably not be erased and newly built for a gradual increase in demand, the charging station will be continuously used in later stages once it is deployed.

Given that the location of charging stations and number of chargers are independent random variables with the $(|K| \times m)$ success probabilities matrix γ, where $|K|$ is the number of nodes in the network and m is the maximum number of chargers that can be located at one station. Corresponding to the charging locations, the vector of feasible paths y then can be identified by the deterministic equivalence of chance constraints on driving range.

$$\gamma = \begin{pmatrix} \gamma_{1,0} & \gamma_{1,1} & \cdots & \gamma_{1,m} \\ \gamma_{2,0} & \gamma_{2,2} & \cdots & \gamma_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{|K|,0} & \gamma_{|K|,1} & \cdots & \gamma_{|K|,m} \end{pmatrix} \tag{13}$$

Accordingly, our problem is to minimize the cost function $PI(X, f(X))$ over all $X(x, u, y)$ in set Ω:

$$z^* = \min_{X \in \Omega} \mathrm{PI}(X, f(X)) \tag{14}$$

The above optimization problem can be associated with an estimation problem:

$$l(z) = Pr\big(\mathrm{PI}\big(X_n, f(X_n)\big) \le z\big) \tag{15}$$

Where X_n is chosen on Ω from a probability density function $f(X, \gamma)$ with sample size N and z is close to (but greater than) z^*. Generally, $l(z)$ is a rare-event probability. As presented in (De Boer, et al., 2005), CEM approach can be used to find an importance sampling distribution so that all its mass concentrates in a neighbourhood of X^*. Therefore, the optimal or near optimal states can be obtained by sampling from such a distribution.

To describe parameterized random mechanism for generating the charging solutions, we consider a solution $X = \big(X_1, \dots, X_{|K|}\big)$ has $|K|$ independent components such that $X_i = j$ with probability $\gamma_{i,j}, i = 1, \dots, |K|; j = 1, \dots, m$. Then, the parameter of sampling distribution at the t^{th} iteration can be updated using following formula \citep{botev2013cross}.

$$\hat{\gamma}_{i,j}^{(t)} = \frac{\sum_{k=1}^{N} I\big\{\big(PI\big(X_k, f(X_k)\big) \le z\big)\big\}\, I\{X_{k,j} = j\}}{\sum_{k=1}^{N} I\big\{\big(PI\big(X_k, f(X_k)\big) \le z\big)\big\}}, \qquad i = 1, \dots, |K|;\ j = 1, \dots, m \tag{16}$$

With the charging solution X, the vector of equilibrium path flow $f(X)$ can be obtained by solving the multi-class probit-based SUE model with Poisson demand. As mentioned above, the multi-class probit-based SUE model based on Poisson-corrected travel time function is adopted to identify route choice probabilities. The link perception errors are assumed to be the same for all vehicle classes and independently distributed for link a as normal distribution, $Nor(0, (\emptyset\, t_a(0))^2)$, with $\emptyset\ = 0.3$ used in numerical tests (Clark & Watling, 2005). The SUE is estimated by using the route-based Method of Successive Average (MSA).

In this paper, we use the convergence between best PI and worst PI during the last two consecutive iterations as the stopping condition for CEM-based algorithm while setting the maximum number of iterations has been reached for the Method of Successive Average. Otherwise, one can stop the procedure when the distance between two consecutive parameter vectors is sufficiently small.

NUMERICAL STUDY

To illustrate the proposed framework's efficacy, the model is tested in a toy network with different scenarios of EVs' driving ranges and charging demand. The network is used by both EVs and conventional gasoline vehicles (GVs) with the aggregate travel demands between O-D pairs as in Figure 2. Link lengths and free-flow travel times are also given in the figure. To gain insights on the impact of EVs driving ranges and EVs market share on the planning decision, we consider different scenarios of EVs' driving range under increasing EVs' penetration. All EVs in the network are assumed to have the driving ranges $D^e \sim Gamma(50,\ 1.5), \mathbb{E}[D^e] = 75$.

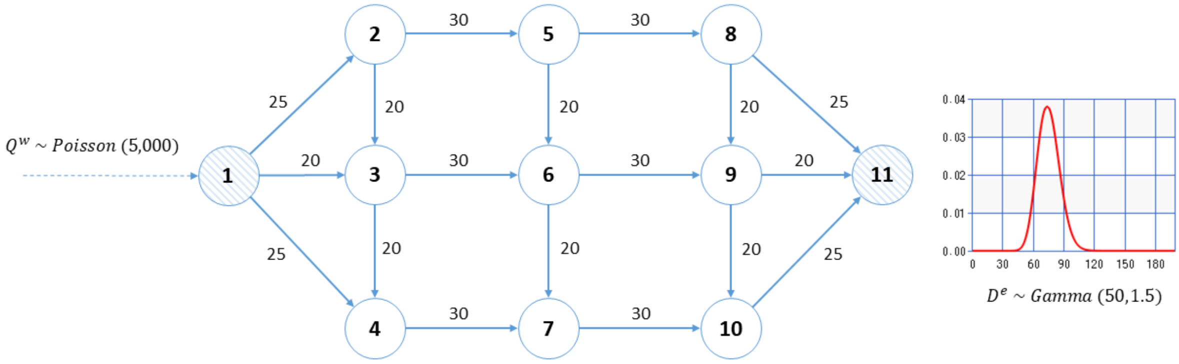

Figure 2. The test-bed Network

Without loss of generality, we assume the link length is the same as free-flow travel time in number which is labelled on each link, and the capacity of each link is 1,800 veh/h/lane. In both numerical tests, the aggregate travel demand between each O-D pair is assumed to be independent stationary Poisson with the mean and variance of 5,000 veh/h. The aggregate travel demands are chosen for the convenience of analysing the impact of en-route and charging congestion.

Lacking the appropriate data, we assume all paths between the O-D pair are feasible for GVs due to their relatively long driving ranges (Jiang, et al., 2014) and the EVs driving ranges follow Gamma distribution due to its flexibility (de Vries & Duijzer, 2017). Maximum acceptable risk threshold in both cases is assumed to be 0.05. Besides, the value of time for all vehicle class is \$20 per hour (Xu, et al., 2017), the cost of opening a new charging station and installing a charger are \$250,000 and \$1,000 respectively regardless of its location (EVSE, 2019).

In the CEM-based algorithm, we first choose the typical sample size in the literature N = 1,000 and the elite sample proportion $\rho = 1\%$. At each iteration, the parameter vector is updated using the smoothing rate $\beta = 0.7$. The stopping condition is zero difference between upper bound and lower bound during the last two consecutive iterations. All instances are solved using Python programming language on a computer equipped with Intel(R) Core(TM) i7-7700 CPU @ 3.60GHz and usable RAM of 15.9 GB, running on Windows 10.

The summary of the optimal location of charging stations, number of chargers and associated costs, i.e. travel cost, environmental cost, investment cost and expected system cost was presented in Figure 3 and Table 2. In general, the expected system cost tends to decrease when EVs' driving range increases while rising sharply when there are more EVs in the network. It can be seen that the investment cost, which results from the planning decision is highly dependent on the traffic pattern on the network. Therefore, the inappropriate deployment of charging infrastructure can increase both congestion and investment cost.

% EVs	Travel time (min)	Waiting time (min)	Travel cost ($)	Environmental cost ($)	Installation cost ($)	System cost ($)
5.0%	660,985.68	3,261.93	221,415.87	106,971.79	1,230,000.00	1,558,387.66
10.0%	642,776.26	7,287.68	216,687.98	98,710.40	1,640,000.00	1,955,398.38
15.0%	648,075.63	12,054.83	220,043.48	94,130.74	2,050,000.00	2,364,174.22

Table 2. The cost of charging facility deployment

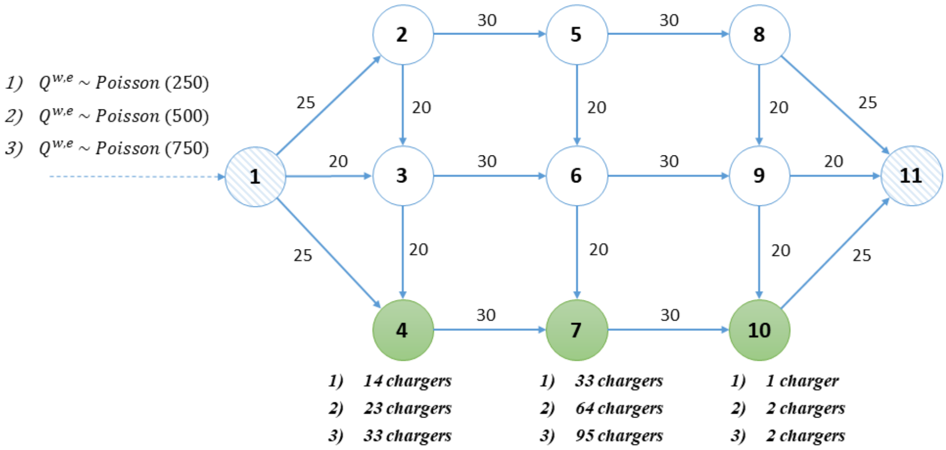

Figure 3. Charging facility deployment solution

Although the electrification of transportation can bring long-term sustainability for urban areas, it is plausible that the network becomes more congested with the increasing EVs' penetration. However, the travel times can be reduced by increasing the number of charging facilities in the network. From an environmental perspective, using more EVs can help to reduce environmental cost caused by the pollutant emitted from GVs. However, environmental cost can increase when there are more EVs. This phenomenon happens because of the on-route congestion in the network. As shown in Table 2, the charging congestion at charging station contributes just a small amount compared to on-route congestion and can be reduced by increasing the driving range of EVs.

Meanwhile, the travel cost depends not only on the driving range and EVs proportion but also on the network's charging facilities. On-route congestion tends to be more serious when EVs' penetration increases, but it can be reduced by improving the EVs' driving range and providing more charging facilities. Therefore, it is worth mentioning that both driving range, the EVs penetration, and the number of charging facilities significantly impact the system performance.

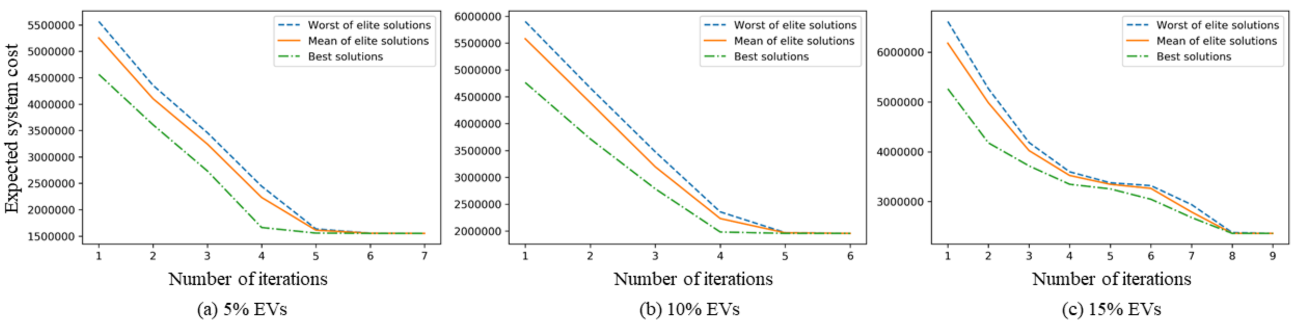

Figure 4. The convergence of planning solutions

Figure 4 illustrates the convergence of PI values in three different scenarios over iterations. The CEM-based algorithm can effectively solve the charging location problem with a larger size network; however, the computational time is increased considerably with the network size.

CONCLUDING REMARKS

In an effort to decarbonise transportation, EVs have been adopted worldwide. However, lacking charging infrastructure poses a significant challenge for the widespread adoption of EVs. In this paper, we proposed a systematic framework to deploy the fast-charging facilities under stochastic driving range, uncertain demand and charging congestion as one of the first attempts to solve the charging location problem considering the highly stochastic nature of the problem.

Although more EVs can bring a cleaner and more energy-efficient transportation system, it sometimes may cause more congestion over the network (i.e., en-route and charging congestion). Therefore, the proposed framework not only minimise the capital cost but also reduce network congestion. The environmental cost caused by GVs is also captured. Numerical tests have shown that the proposed CEM-based approach can provide a good solution for such a complex problem.

However, the computational cost remains a burden, especially for large-scale networks. Besides, the bi-level optimisation framework is a non-linear and non-convex optimisation problem in which there is no single approach to obtain a general global optimal solution. We leave the problem of reducing the computational cost for future study, which is part of the authors' ongoing research. The proposed can also be developed to capture the operational constraints (e.g., time windows) to apply for electric buses.

AUTHOR CONTRIBUTION STATEMENT

Cong Tran carried out the primary research under the supervision of Drs. Mehdi Keyvan-Ekbatani and Dong Ngoduy.

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